THEORY OF INTERNATIONAL VALUES[1]

II

THE mathematical version of the theory consists either of Geometry or Algebra.

Geometry is directly applicable only to the simplest possible cases. If more than two commodities are considered, solid geometry must be called in. The dimensions of space are not adequate to represent the case of more than three variables.

Geometry therefore might appear to have no application to reality; since countries importing or exporting only one article exist only in imagination. But the geometrical representation of this imaginary case is useful as suggesting theorems which may be seen to admit of extension to more concrete cases.

The simplest geometrical representation of international trade appears to be a construction first used by Professor Marshall and explained by him in the mathematical appendix to his *Principles*.[2]

In Figure 1, the curve O E, which might be called England's Supply-and-Demand curve, signifies that for a certain quantity O x of English produce, say 'cloth,' exported, the quantity O y of German produce is demanded. The supply of linen and demand for cloth on the part of Germany are similarly expressed by the curve O G.

With respect to these curves it is not, I think, necessary to make the supposition which is usually made with respect to more familiar demand or supply curves—namely, that while the rate of exchange represented by the curves is varied, the rate of exchange between one of the ordinates and all other articles—the price of all other articles, as it would usually be expressed—remains constant.[3] Rather a movement along a supply-and-demand curve of international trade should be considered as attended with

[1] See the first article published in the ECONOMIC JOURNAL for March 1894; hereinafter referred to briefly as *ante*.

[2] Note 12, second edition.

[3] *Cf.* Auspitz and Lieben, *Theorie der Preise*, pp. 4, 155, &c.; Cournot, *Principes*, ch. xi. Art. 74; Marshall, *Principles of Economics*, Book III. ch. iii. § 6.

rearrangements of internal trade; as the movement of the hand of a clock corresponds to considerable unseen movements of the machinery. Accordingly, the marginal utility of imports need not be supposed constant [1]; nor the marginal disutility, the cost of production, of exports.[2]

The theory of comparative costs is not very prominent from the mathematical point of view.[3] It may be represented geometrically as follows. Let the cost of production at first be supposed

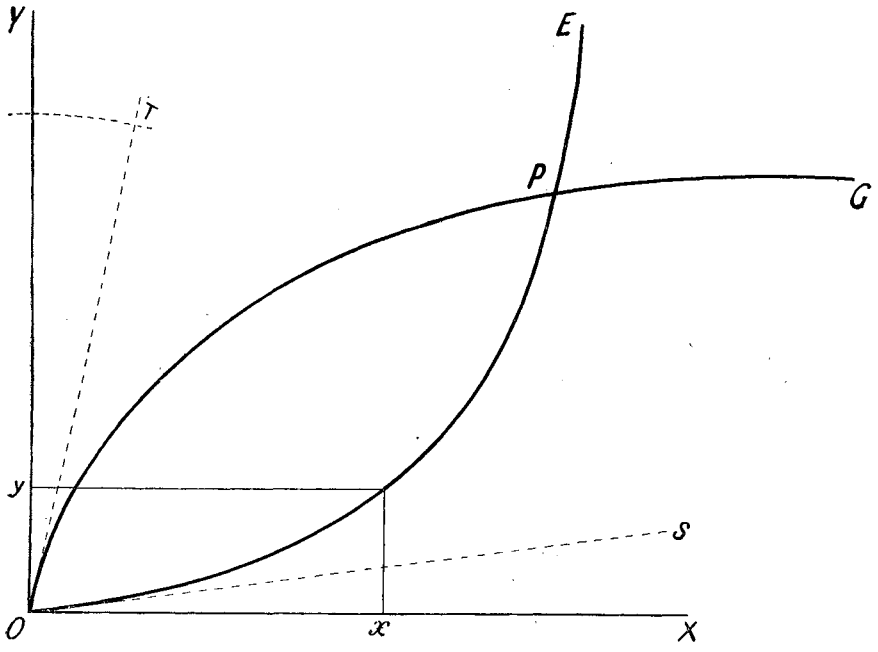


FIG. 1.

constant; then the terms on which England could have obtained linen in the absence of the trade may be represented by a straight line O S, if tan. S O X = ratio of the cost of production of a unit of linen to that of a unit of cloth. In order that England may obtain linen cheaper with than she could without the trade, the point of equilibrium must be *above* the line O S. It must be below the line O T, in order that Germany may be benefited. To generalise this theory there should be substituted for the straight line O S (and *mutatis mutandis* for O T) a curve of constant

[1] As by Messrs. Auspitz and Lieben when they take money of constant marginal utility as the import.

[2] As by J. S. Mill.

[3] *Cp.* Pareto, 'Cambi Forestieri,' *Giornale degli Economisti*, 1894, p. 154.

advantage, or 'indifference-curve' (not shown in the figure), representing states for which the advantage to England is no greater than if there had been no trade.[1] That the point of equilibrium falls between the respective indifference-curves is the geometrical version of Comparative Costs. The expression which occurs in some of the best writers, that international value 'depends on' comparative cost, is seen from this point of view to be a very loose expression.[2]

In investigating the incidents attending differences in the conditions of supply and demand[3] it is important to distinguish the varieties of data. This purpose may be assisted by the following logical tree, or ramification; where the capital letter corresponds to a positive, the small Roman to a negative attribute.

A, International trade proper; a, quasi-international trade (in particular, distribution).

B, the case of two nations only; b, of several.

C, where we regard the interest of only one, our own, country; c, where we regard the interest of all parties concerned.

D, where we regard present advantage only; d, future also.

E, where we are concerned only with functions of the simple form proper to 'short periods'[4] (such as the curves in Fig. 1), and accordingly the changes contemplated are in a sense small;[5] e, where more complicated functions and organic changes[6] are considered.

F, where the change considered originates in a foreign country; f, in the home country.

G, an improvement or impediment other than a bounty or tax; g, a bounty or tax.

H, where the change originates on the side of supply: such as increased facility of producing or exporting native commodities; h, on the side of demand: such as an increased desire for, or facility in admitting, foreign commodities.

By ringing the changes on these positive and negative attributes some hundreds of different cases can be distinguished; thus (1) A B C D E F G H, (2) A B C D E F G h, (3) A B C D E F g H, (4) A B C D E F g h; and so on up to 2^8.

[1] See the present writer's *Mathematical Psychics*, pp. 21–29.

[2] No doubt, as Professor Bastable has pointed out, when there are numerous competing nations, the limits fixed by the principle of Comparative Cost are much narrowed; and accordingly it becomes less incorrect to regard the principle as sufficient to determine international value.

[3] As proposed *ante*, p. 37, par. 2.

[4] Marshall, *Principles of Economics*.

[5] *Ante*, p. 38.

[6] Described below, p. 436.

But of the compartments thus formed many would be empty, such as those which combine c, regard for the interest of all nations, with F or f, distinguishing natives and foreigners. It is proposed to consider only the more important cases—namely, those which have been summarily treated in the preceding article.

A B C D E F.[1]—This is the case of international trade proper, between two countries, regard being had to the interests of the home country only, and immediate or direct effects only being considered; and a certain simplicity in the law of demand and supply for both countries being assumed, a change is supposed

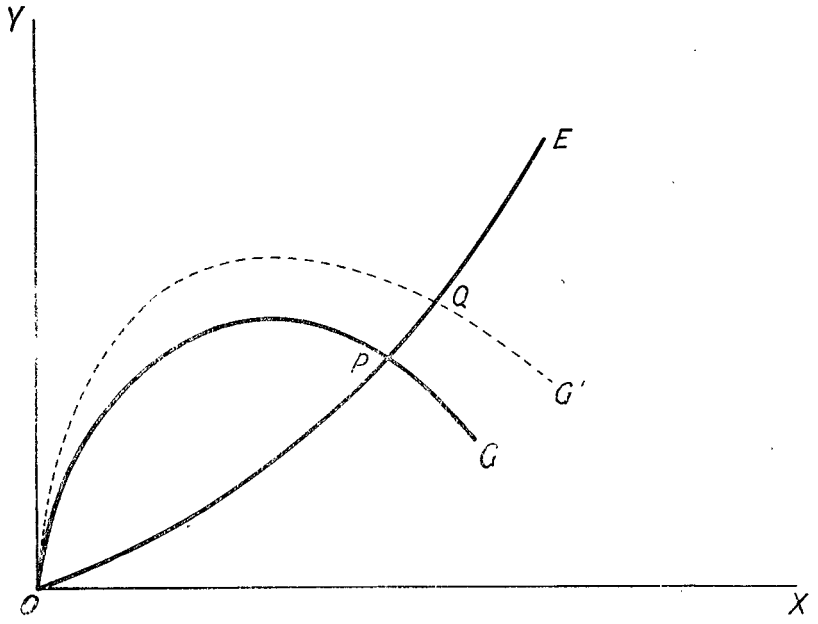


FIG. .

to occur in the terms on which the foreigner is willing to trade.

The increase of the supply of foreign produce (in the sense that more of it is offered at each rate of exchange) is represented in Fig. 2 by the displacement of the foreign curve O G to O G'. Whatever the direction[2] of the native or the foreign curve in the neighbourhood of their intersection, it will be found that in every case the new intersection has travelled along the native curve *away* from the origin. Whence the change is bene-

[1] *Ante*, p. 38, last par.

[2] Consistent with the condition that the equilibrium should be stable.

ficial[1] to the native country. Conversely, a diminution in the offer of foreign goods is prejudicial to the home country; as may be seen by taking the dotted curve as the original one.

A B C D E f G H.—The case of f, a change originating in the home country, is not so simple.[2] The answer varies according as the letters, after f, are capital or lower case, designate positive or negative attributes. In each case much turns upon what Mill calls the extensibility of demand.[3] This property may be thus contemplated. Draw a line parallel to the

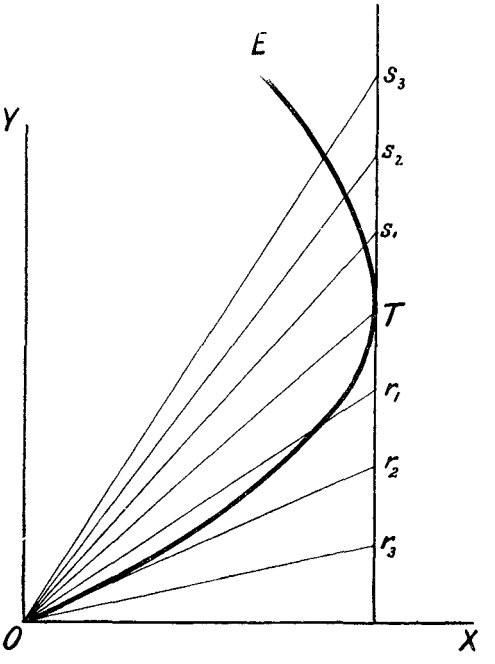


FIG. 3.

axis Y touching the curve O E in T (Fig. 3). Divide this line into a number of equal small parts: Tr_1, r_1 r_2 below T, and Ts_1, s_1 s_2 above T. Each interval corresponds to an increment in the value of X with respect to z, that is the number of units of z given in exchange for a unit of H. Join r_1, r_2, &c., s_1, s_2, &c. to O; and from the points r_1, r_2, &c., s_1, s_2, &c., let fall perpendiculars—not shown in the figure—on the axis Y. Then it appears that below the point T an increment in value of

[1] If this proposition is not self-evident, I may refer for a proof of it to my *Mathematical Psychics*, p. 115.

[2] *Ante*, p. 39, last par.

[3] Book III. ch. 18, s. 1.

X corresponds to a more than proportionate increase in the quantity of Y demanded; and conversely, above the point T. We may describe the curve above T as *elastic*,[1] below inelastic. Each of the cases comprised under A B C D E f are divisible into four subcases, according as the native or foreign curve is elastic or inelastic.

A B C D E f G H I.—This is the case of a decrease (or increase) in the supply of exports due to a cause other than the imposition (or remission) of a tax: such as a change in the cost of production, or transport.[2] The four subcases are represented by the four varieties of Fig. 4; O E being as before the native curve, and O E′ what it becomes by the change considered.

Subcase (1) is where both native and foreign curves are elastic. The native curve O E becomes transformed by the impediment to O E′. In the new equilibrium indicated by the point Q, R Q of X is given in exchange for Q S of Y. But Q cannot be a position of greater advantage than P′, where the horizontal through Q cuts the original curve. For, on the most favourable supposition that the impediment affects only exportation, not production for internal consumption,[3] England's offer in exchange for O R would be reduced by the impediment from O S′ to O S, so that Q would be a position of just equal advantage as P′. But P′ is a position of less advantage than P (being nearer the origin as you move along the curve). Thus the native country is prejudiced by the change.

The converse variety of the subcase, where an improvement, not an impediment, has supervened, may be investigated by treating OE′ as the original, OE as the displaced curve. Whence it appears that the native country is advantaged by the change.

In subcase (2), where the native curve is elastic, the foreign inelastic, by a parity of reasoning the natives may be benefited by an impediment, and prejudiced by an improvement.[4]

In subcase (3), where the native curve is inelastic, the foreign elastic, the natives are prejudiced by an impediment and benefited by an improvement, as in subcase (1).

In subcase (4), where both curves are inelastic, the natives may be benefited by an impediment and damaged by an improvement, as in subcase (2).

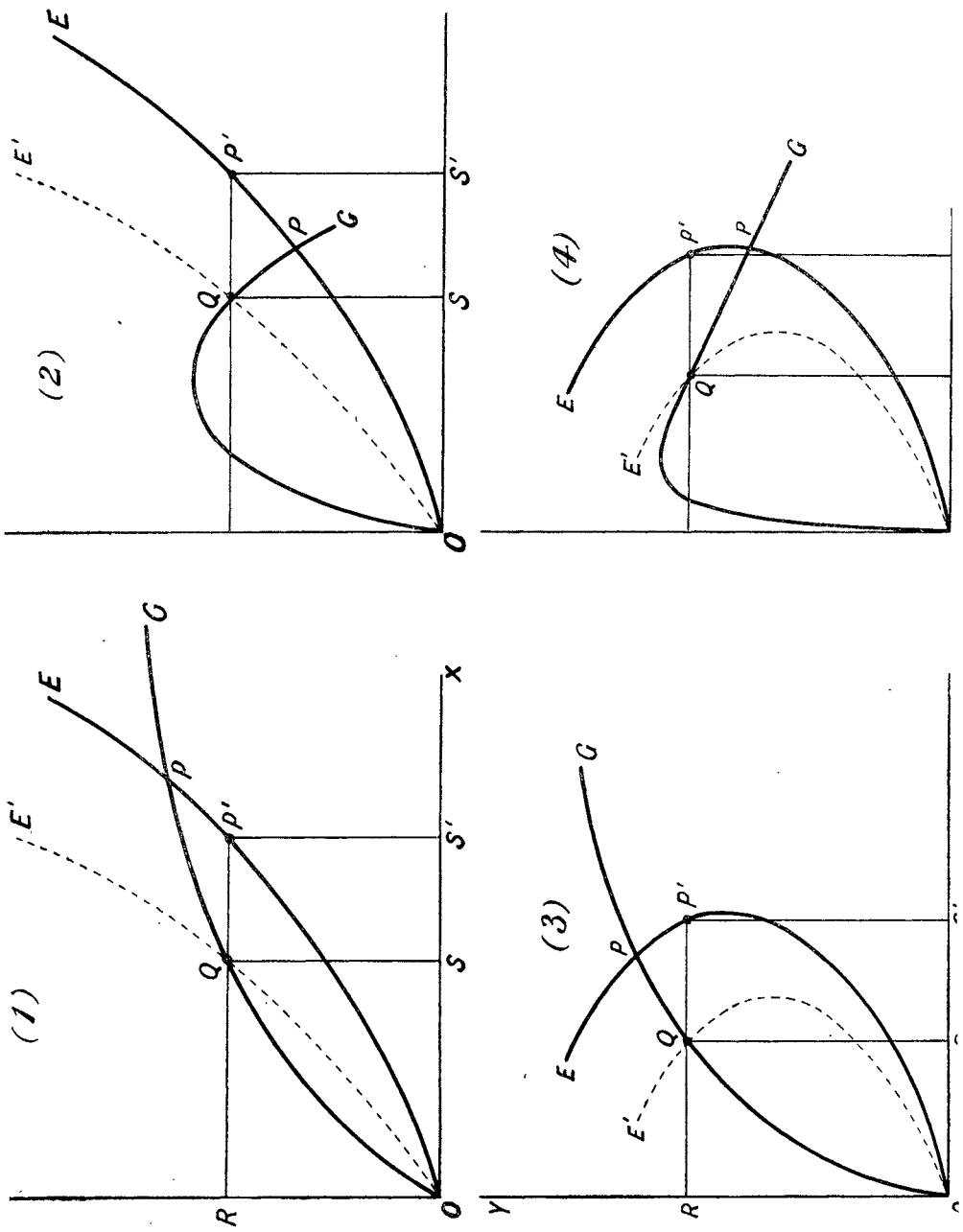
These results may be summed up in the diagram forming Fig. 5, which shows the consequences of an impediment;

[1] *Cf.* Marshall, *Principles of Economics*, Book III. ch. 4.

[2] *Ante*, pp. 40, 41.

[3] For instance, a transit duty imposed by a third country. *Ante*, p. 41, par. 2.

[4] *Ante*, p. 41, penultimate par.


E
E'
P'
P
G
Q
S'
S
O
(2)
(4)
(1)
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X
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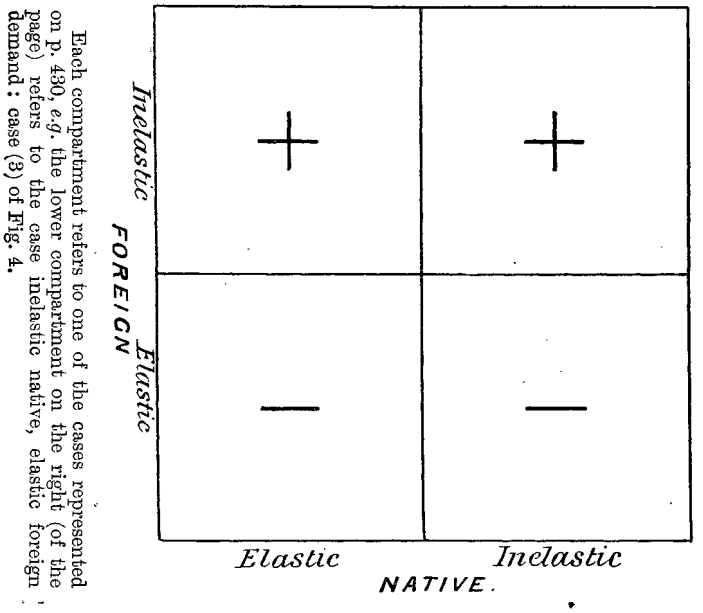
the symbol + denoting advantage to the natives *ceteris paribus*, or abstracting the effects on internal trade; the symbol − denoting disadvantage without qualification. To exhibit the consequences of an improvement converse signs should be used.



Each compartment refers to one of the cases represented on p. 430, *e.g.* the lower compartment on the right (of the page) refers to the case inelastic native, elastic foreign demand: case (3) of Fig. 4.

FIG. 5.

A B C D E F G h.—In the case of an impediment affecting imports, the displaced curve is formed by lengthening the ordinate instead of shortening the abscissa of the primary curve. Where the native curve is elastic, that is in subcases (1) and (2), the same figures will serve for h as for H. But in subcases (3) and (4) the diagrams forming Fig. 6 must be substituted for those which are proper to case H. As in case H, *mutatis mutandis*, the consequence of an impediment may be represented by the change from P to P′; those of an improvement from P′ to P (see Fig. 6).

The diagram on p. 432 (Fig. 7), corresponding to that on this page, shows the consequences of an impediment to importation. The consequences of an improvement are obtained by changing each of the signs. It will be observed that a restriction on exports is not so certain to be prejudicial to the country imposing it as one on imports.

A B C D E f g H.—The case of a tax[1] differs from that of an

[1] *Ante*, p. 42, last par.

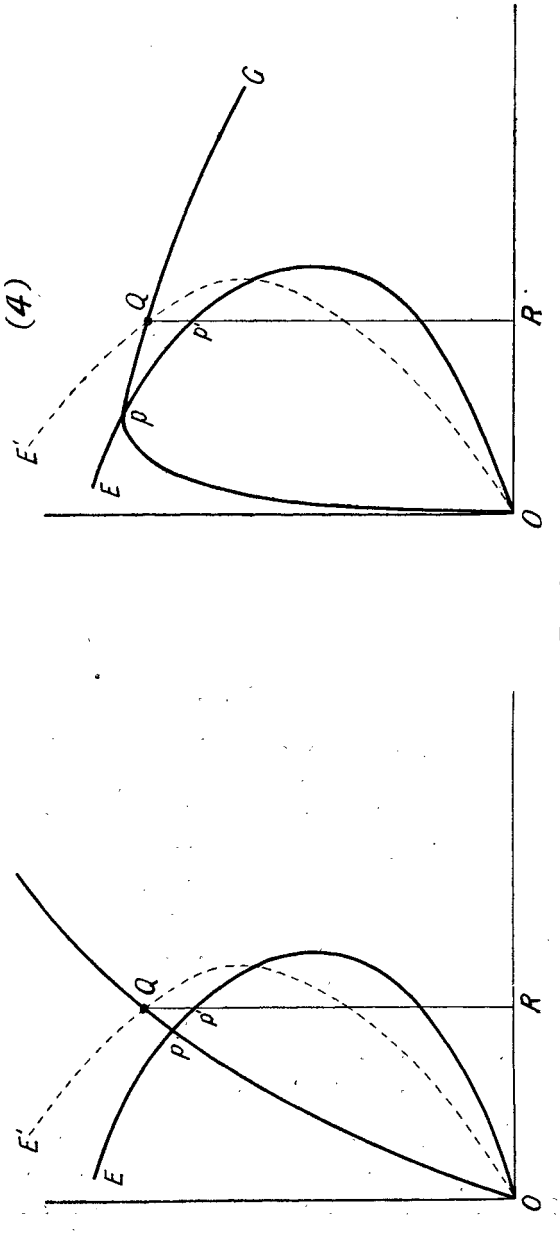
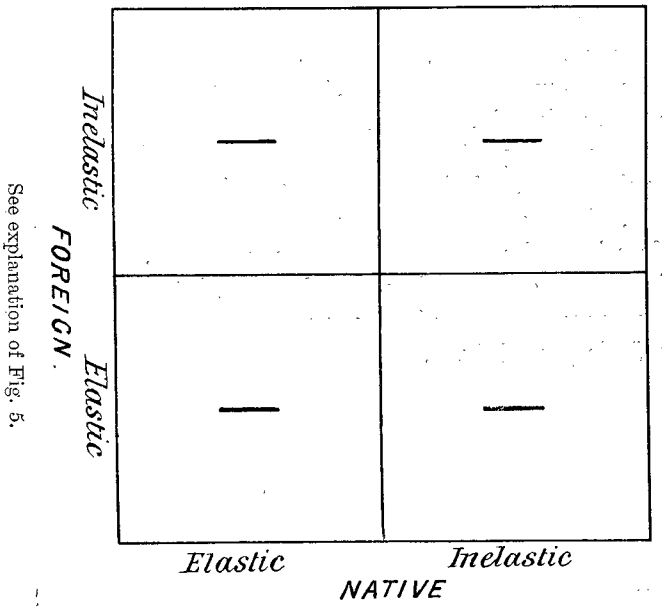


FIG. 6.

impediment in that the change is not now from P to P', but from P to Q. To consider whether this change is advantageous or



See explanation of Fig. 5.

FIG. 7.

not we may employ the conception of an *indifference-curve* or locus of positions of trade which are of equal advantage as any

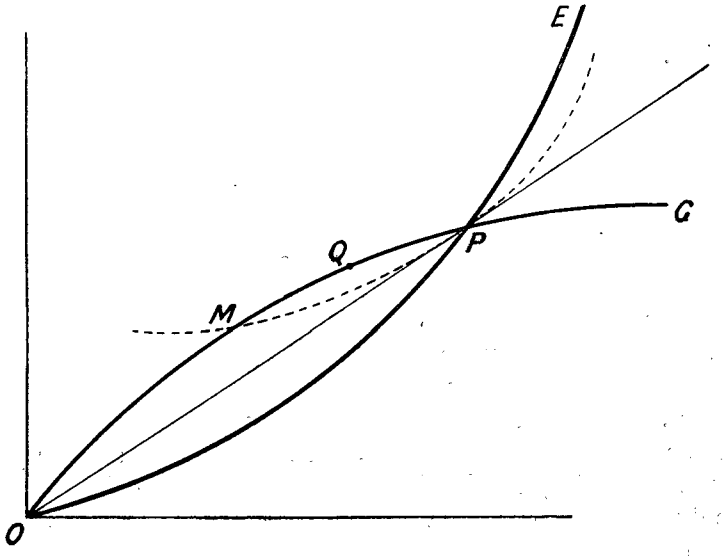


FIG. 8.

assigned position P.[1] P being on the supply-and-demand curve O E, it may be shown that the indifference-curve touches the vector from the origin to that point, O P in Fig. 8.

Let the native indifference-curve through P cut the foreign demand-curve O G in M. Then, if Q, the new position of equilibrium, on the curve O G (see Fig. 8), is above M, *inside* the indifference-curve, as in Fig. 8, the natives are benefited; if Q is below M the natives are prejudiced. In the subcase illustrated by Fig. 8, viz. subcase (1), it is in general uncertain whether Q is above or below M. The consequence represented by the sign − in the case of an impediment (Fig. 5) becomes now ±. But the sign + (in subcases (2) and (4)) becomes *à fortiori* +.

These consequences of a tax on exports are exhibited in the annexed diagram, Fig. 9.

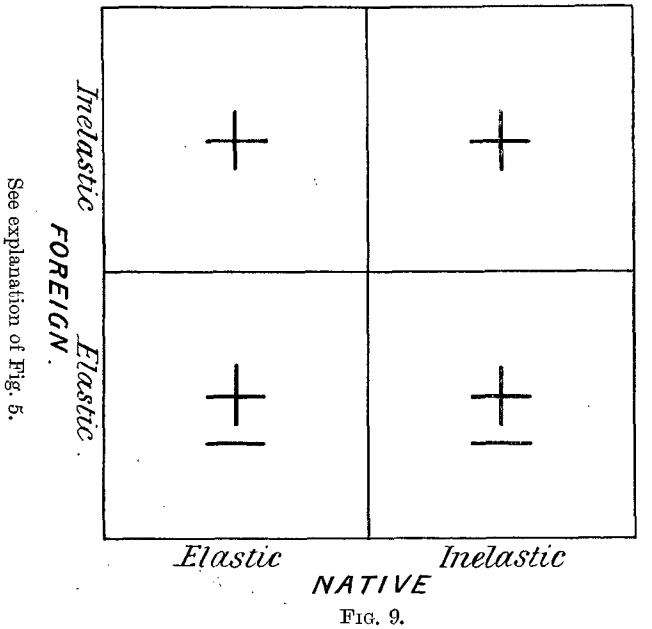


FIG. 9.

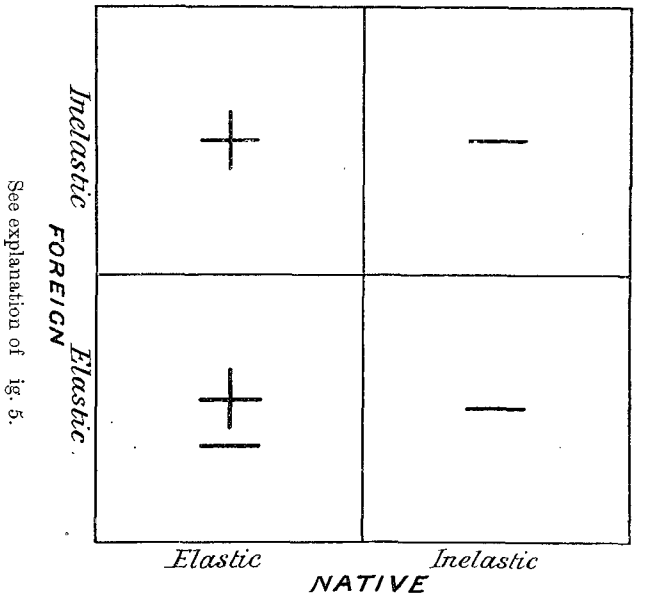
From the reversibility of the positive and negative cases, which has so far prevailed, it might have been expected perhaps that a bounty should be advantageous in cases where a tax is detrimental. But this analogy is misleading; so long, at least, as we confine ourselves to attribute E.[2] The action of a bounty may be represented by considering in Fig. 4, Q as the original, and P the displaced point. In every subcase it will be found that the

[1] *Mathematical Psychics*, p. 21.

[2] *Ante*, p. 48, par. 2.

displaced point is on the *right* of the line joining the origin to Q; that is, *outside* the indifference-curve which touches the line at Q on the *other* side.[1] (*Cf.* Fig. 8.)

A B C D E f g h.—The consequences of a tax on imports, investigated by parity of reasoning, are represented in the diagram which forms Fig. 10.



See explanation of fig. 5.

FIG. 10.

Comparing this diagram with the preceding, we infer that a nation is more likely to benefit itself by an export than an import tax.[2]

The want of symmetry between the effects of restrictions, and in particular taxes, on exports and imports, is perhaps the conclusion which can be most peculiarly and exclusively attributed to the mathematical method. The truth does not seem to have been clearly attained by the ordinary methods.

A B C e.—So far we have supposed the curves O E and O G to be of the simple form shown in Fig. 1. In considering complicated re-entrant forms like that in Fig. 11, it will be convenient to begin by restoring the usual supposition that the marginal utility of one of the commodities is constant. Thus let us for a moment

[1] This conclusion agrees with that of Messrs. Auspitz and Lieben, as was to be expected, their curves being of our Class E. (See *Theorie der Preise*, p. 425.)

[2] *Ante*, p. 43.

regard O E as a supply-curve indicating that a certain quantity of cloth O x is supplied in exchange for a certain quantity of a commodity whose marginal utility may be regarded as constant, say money.[1] Then two kinds of supply-curve may be distinguished, (I) representing the amount of cloth which would be offered at each price, no account being taken of the change in the offer due to the alteration in the scale of production for different values of the *primary* supply-curve, as we may call it. It seems to be much the same as Professor Marshall's short period supply-curve. (II) Next let us take account of the change in the offer due to alteration in the scale of production; and so form a series of primaries corresponding to each value of x: Mr. Cunynghame's 'successive cost-curves.'[2] If now at each point on the abscissa an ordinate is erected, the *locus* of intersection with the corresponding 'successive cost'-curve forms a *secondary* supply-curve: Mr. Cunynghame's supply-curve; and, as I understand, Professor Marshall's 'long-period' supply-curve.

It is a nice question whether a primary cost-curve can be regarded as re-entrant in the manner represented in Fig. 11.[3] Perhaps we may with sufficient generality consider that it cannot. The secondary curves are (*a*) sometimes of the simpler form represented in the earlier figures; (*b*) sometimes re-entrant as in Fig. 11. Curves of the former kind, that is IIa, have many properties in common with species I;[4] in particular that movement along the curve in a direction from the origin is attended with advantage.

We have just been regarding O E as a supply-curve. Now let us regard it as a demand-curve in this sense that O y linen (see Fig. 1) is demanded in exchange for O x of a commodity whose marginal utility is constant, say money. Then from this point

[1] As in Messrs. Auspitz and Lieben's constructions.

[2] ECON. JOURN., vol. ii.

[3] As argued by the present writer elsewhere (Address to Section F of the British Association Report, 1889, Note J). Though at a given rate there may be several *maxima* of advantage, there can be only one position of *greatest possible* advantage. Since, then, the motive of the economic man is greatest possible, rather than merely maximum advantage, it should seem that the ordinate of the supply-curve corresponding to each value of tan. P O X must be unique; discontinuous for the individual who must be conceived as jumping from one branch to another when a certain value of tan. P O X is reached, but continuous for the community since the point of transition will be different for different individuals. On the other hand, there may exist friction obstructing the movement from a small to a large scale of production; and so two branches of the curve exist simultaneously. In this case, as pointed out by the present writer (*Mathematical Psychics*, Appendix 7), the tract between T and T'—points where tangents drawn from the origin touch the curve—is not a genuine demand-and-supply curve, being a locus of *minimum* advantage.

[4] Marshall, *Principles of Economics*. Note to p. 484, 2nd edition.

of view also, if the law of demand is considered to vary with the scale of consumption, as Mr. Cunynghame supposes, the curve may prove to be re-entrant.[1] I submit, however, that this cause of abnormality is less important and less capable of being formulated than the influence of the scale of production on cost.

Not that from either point of view an exact determination of the curve is to be expected; we must be content with general descriptions: such as elastic and inelastic, re-entrant or not. Still less definiteness is attainable when, combining the two views

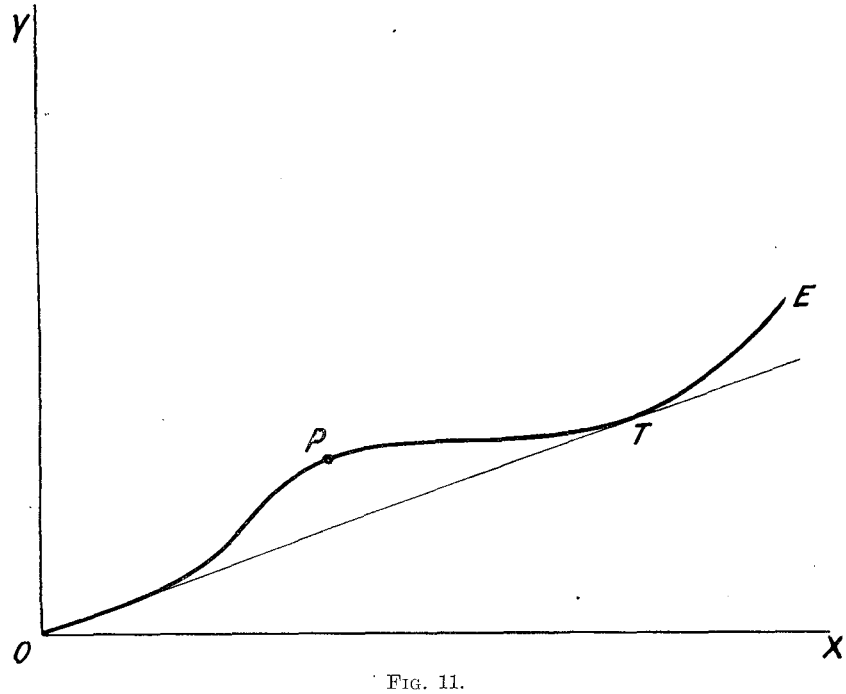


FIG. 11.

which have just been distinguished, we restore our original view of the demand-and-supply curve O E: as representing the interchange of two articles of variable marginal utility.

The consequences of the property of re-entrance may be considered under the head (d), which indeed is with difficulty separated from (e); since, in fact, organic changes only occur in long periods.

A B C d.—Many of the propositions, stated under preceding heads, no longer hold when we consider organic changes extending over long periods. Thus it ceases to be universally

[1] Ascending in Mr. Cunynghame's construction. See ECON. JOURN., vol. ii.

true that an increase in the supply of foreign commodities is beneficial to the native country.[1] For the curve O G being shifted upwards might strike the native curve in the neighbourhood T (Fig. 11), corresponding to a lower value of the native produce with respect to the foreign,[2] and a lower value of the native goods may be attended with detriment to the native country.[3]

Again a bounty ceases to be universally disadvantageous. For, in the manner shown by Professor Marshall with respect to a different construction, a bounty may shift the point of equilibrium to a position more advantageous to the community.

A B c D E.—When we consider the interest of both parties, not of one only, the chance of benefit resulting from interferences with trade is diminished. The presumption that any such interference impairs the total utility is well illustrated by Messrs. Auspitz and Lieben, on the tacit assumption that what may be called the hedonic worth of money is the same in both countries. The generalised form of that assumption—appropriate to our system of co-ordinates, which does not represent money—is that if for each party a curve be drawn cutting at right angles the system of indifference-curves—called by the present writer a *preference-curve*—the same distance along such a curve corresponds to the same increase or diminution of advantage on both sides. This is no doubt an allowable assumption, in the absence of knowledge to the contrary. But, when we know that one party is much better off than another,[4] the assumption may be illegitimate.[5]

A B c d e.—The doctrine that interferences with trade are detrimental to the community of nations becomes more questionable when we consider organic changes operating for a consider-

[1] Above, p. 427.

[2] The proposition set forth in the books (*e.g.* Mill, *Political Economy*, Book III., ch. xviii.) that the setting up of trade is advantageous to both countries assumes that the curves [or the analogous algebraic functions in the general case] with which we have to deal are of the form I on II (*a*). In that case the position of stable equilibrium may be regarded as a point of maximum of advantage in excess of the *adjacent* minimum formed by the position of null trade, viz. the origin. But, if curves of the form II *b* prevail, then a position of stable equilibrium, though a maximum, may be attended with less advantage than the position of null trade.

[3] *Ante*, p. 48.

[4] This is most likely to occur, I think, in the quasi-international trade between the parties to Distribution.

[5] This, if not already evident, may be contemplated by regarding the contract-curve as the locus of points (*Mathematical Psychics*, p. 21 *et seq.*) at which the preference-curves of the two parties coincide with opposite directions. According to the assumption in question, it would be indifferent, from the point of view of the general good, whether all the advantage of trade accrued to one party, or both had a share.

able time. The possibility that such measures should be attended with advantage to all is well shown by Professor Sidgwick in his chapter on protection.[1]

A b C D E F.—The case of trade between several nations which lends itself best to geometrical illustration is that of a third party competing with the home country, as we may call that one whose advantage is exclusively regarded, for trade with foreigners.

In Fig. 12 let O G be the foreign curve, O e the native, O ϵ

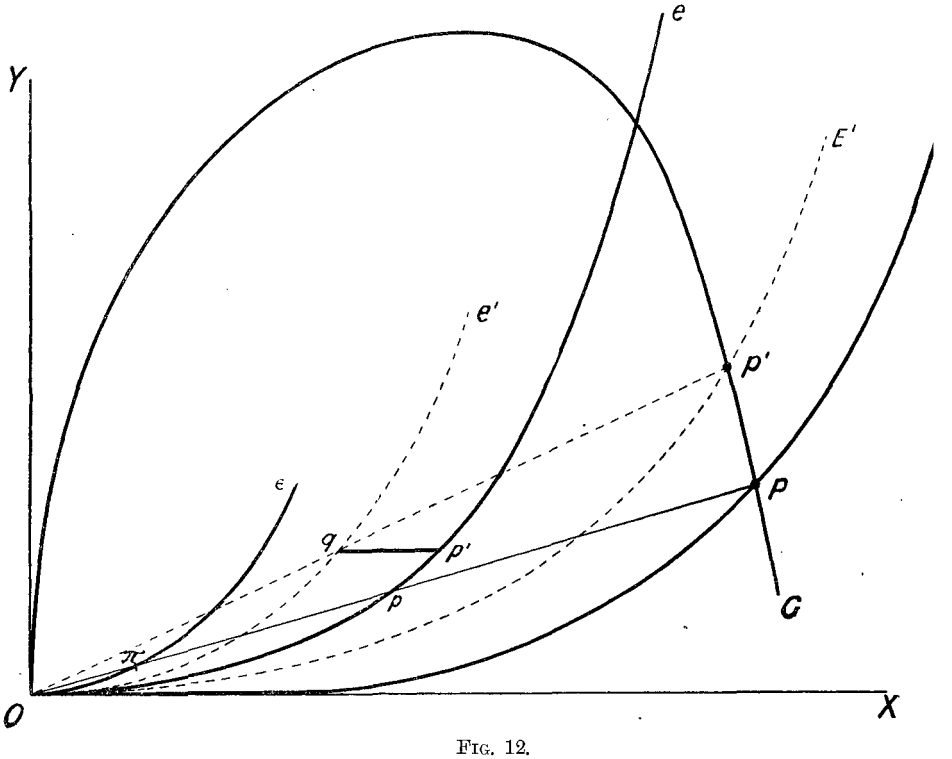


FIG. 12.

the competing, and O E compounded of the last two. The detriment inflicted on the home country by the competition may be described as the change in a backward direction along the curve O e from the intersection of O e with O G to p, where the line O P cuts O e.

It is to be observed that competition does not necessarily deprive a country of the advantage which it may derive by a restriction of imports. For suppose that in the absence of competition the conditions described in subcase (2) of A B C

[1] *Ante*, p. 49, par. 2.

existed, so that the home country could benefit itself by a restriction of imports; then after the rise of competition it may still be possible for the home country to benefit in the way described under the heading referred to.[1] The restriction will transform O e to O e′, O E to O E′, P to P′, p to q, which is apt to be a position of equal advantage as p′, and therefore of greater advantage than p. It may be observed that this species of benefit to the home country may be made possible by competition, not having been so before, if O G is inelastic at its intersection with O E, but not at its intersection with o e.

a.—The incidents of quasi-international trade—*e.g.* between the parties to Distribution—do not lend themselves to geometry so well as to algebra, on which we now enter.

In entering upon the more complicated part of the subject, it is well to recall Professor Marshall's warning words: 'When a great many symbols have to be used, they become very laborious to any one but the writer himself,' and 'it seems doubtful whether any one spends his time well in reading lengthy translations of economic doctrines into mathematics, that have not been made by himself.'[2] It is easier to strike out a new path for oneself than to plant one's steps in the footprints of another.

It is almost sufficient to indicate the general scope of the inquiry—namely, to determine that state of trade for which the sum of the utilities of all parties concerned regarded as functions of the amounts of commodity consumed, less the sum of the disutilities regarded as functions of the amount produced, is a maximum; subject to the conditions that what is bought is sold, what is consumed is produced, the 'law of indifference,' the existence of non-competing groups, and so forth.[3]

A few more particular directions may be added.

Let us begin with the case next in point of simplicity to that which has been treated: where there are two countries, one of which exports two articles, the other one article. Let x and y be the amounts of the two articles exported by the first country, and z the amount of the article exported by the second country. Let X and Y be the amounts of the articles produced in the first country which are consumed in that country, and Z the amount of the article produced in the second country which is consumed in that country.

[1] *Ante*, p. 42.

[2] Preface to *Principles*, 1st edition.

[3] *Cf.* Marshall, *Principles*, 2nd edition, note xii.; and the formulæ given by the present writer in the notes to the Address to Section F of the British Association (1889).

Let us first consider the abstract case in which the cost of production is constant; say a_1, a_2 units of work[1] in the first country go to a unit of each of its two products respectively; b_1 units of work in the second country to a unit of its product. Suppose also at first the number of units of work available to be a fixed quantity, say A and B, in the two countries respectively. Then we have

$$(1)\quad \begin{cases} a_1(x + X) + a_2 (y + Y) = A \\ b_1(z + Z) = B. \end{cases}$$

The advantage of the first country which is to be maximised, subject to the first of the above-written conditions, and the corresponding advantage of the second country, may be written—

$$(2)\quad \begin{cases} \Phi (X, Y, z) \\ \Psi (z, y, Z). \end{cases}$$

The position of equilibrium is determined by the values of the variables which make each of the above-written expressions a maximum; subject to the conditions stated by equations (1), and to the further condition—

$$(3)\quad a_1x + a_2y = vb_1z;$$

where v is the rate of exchange between the product of work in the two countries, the number of units of work in the first country, of which the product is equivalent to the product of a unit of work in the second country.

That Φ and Ψ should each be a maximum, subject to equation (1), may be expressed by proposing each of the following expressions to be maximised—

$$(4)\quad \begin{cases} \Phi (X, Y, z) - \lambda [a_1(x+X) + a_2(y+Y) - A]; \\ \Psi (x, y, Z) - \mu [b_1(z+Z) - B]; \end{cases}$$

where λ and μ are indeterminate factors.

The expressions (4) become by equation (3)—

$$(5)\quad \begin{cases} \Phi (X_1 Y_1 z) - \lambda [a_1 X + a_2 Y + vb_1 z - A]; \\ \Psi (x, y, Z) - \mu \left[\frac{1}{v} (a, x + a_2 y) + b, Z - B \right]. \end{cases}$$

Differentiating the first of these expressions with respect to X, Y, z respectively, and the second of the expressions with respect to x, y, Z respectively, we have six equations, which with the pair of equations (1) and the equation (3) make nine equations, to determine the nine unknown quantities, x, y, z, X, Y, Z, λ, μ, v.

[1] More exactly 'effort and sacrifice'; involving at least two dimensions of disutility, labour and waiting.

Eliminating the last six of these variables, we obtain three equations of the form—

$$(6)\quad \begin{cases} \Phi_1 (x, y, z) = 0 \\ \Psi_1 (x, y, z) = 0 \\ \Psi_2 (x, y, z) = 0 ; \end{cases}$$

which are the analogues of the demand (and supply) curves proper to the case of two commodities; *e.g.*, Φ_1, giving the amount of imports demanded by the first country in exchange for assigned amounts of export, y and z. The position of equilibrium may be regarded as the intersection of the three surfaces designated by equation (6).

Conclusions analogous to those which have been obtained for the case of two commodities are easily discerned to be obtainable in the case of three or more variables. Thus, if the second country has an urgent demand for one of the commodities, say x, of the first country, it is possible that an export tax on x may be beneficial to the first country; while an export tax on y might not have that effect.[1]

I do not know that any fresh conclusions are presented by the case of many variables. Accordingly it may be left to the reader to elaborate that case.[2] It will be sufficient here to indicate how some of the concrete circumstances which have been abstracted may be restored.

First, the cost of production may be treated as varying with the amount produced by regarding a, x [a_2, x, &c] not as the product of x by a constant, but as a definite function of x.

Again, the amount of work available may be treated as variable, by regarding A [B, &c.] not as a constant, but as a function of disutility, which disutility is to be subtracted from Φ [Ψ, &c.] in order to obtain the expression which is to be maximised.[3]

The cost of transport may be introduced by regarding the littoral of one country as the scene of the market, and treating

[1] *Cf.* above, p. 434.

[2] A statement of the general case is given by Professor Pareto in his able article on 'Teoria Matematica dei Cambi Forestieri,' in the *Giornale degli Economisti*, 1894, Art. 9 *et seq.*

It may be observed that the formulæ given by Professor Pareto (in the earlier part of his article), after Professor Walras, as proper to the case of industrial competition (domestic trade), are also applicable to that case of trade between 'nations (or 'non-competing groups') in which each commodity is produced by only one nation. The formulæ do not express the essential attribute of domestic trade, viz. the tendency to equality in the net advantages of different occupations. Such net advantages, being of the nature of total utility, could not be expressed by formulæ involving only final utility. This is the gist of my criticism of Professor Walras, to which Professor Pareto replies in the article referred to (*loc. cit.* p. 144).

[3] *Cf.* Marshall, *Principles*, App ndix, note xii. 2nd edition.

the cost of importing foreign articles thereto as part of the cost of production.[1]

It is unnecessary to show how the number of commodities and number of countries may be multiplied. What Mill says of the theory of value in general is particularly applicable to the mathematical version of it: 'The further adaptation of the theory . . . may be left with great advantage to the intelligent reader.'

It remains only to acknowledge my obligation to Professor Marshall's unpublished chapters on foreign trade. These are the chapters alluded to in the Preface to the *Principles of Economics* as having been printed for private circulation and sent to many economists. Part of their substance is contained in the first volume of the *Principles;* part may be looked for in the second volume. What is written on the subject after a perusal of the privately circulated chapters, and pending the publication of the second volume, can make no claim to originality or permanence—like the light of the planet which precedes the rising of the sun, borrowed from and destined to be effaced by the prime orb.

F. Y. EDGEWORTH

[*To be continued.*]

[1] *Cf.* Pareto, 'Cambi Forestieri,' *Giornale degli Economisti*, p. 153.

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