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# Term Structure Dynamics and No-Arbitrage under the Taylor Rule

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Term Structure Dynamics and No-Arbitrage under the
Taylor Rule

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To my family.

# Acknowledgements

Abstract

The term structure interest rate determination is one of the main subjects of the financial assets management. Considering the great importance of the financial assets for the economic policies conduction it is basic to understand structure is determined. The main purpose of this study is to estimate the term structure of Brazilian interest rates together with short term interest rate. The term structure will be modeled based on a model with an affine structure. The estimation was made considering the inclusion of three latent factors and two macroeconomic variables, through the Bayesian technique of the Monte Carlo Markov Chain (MCMC).

Key words: term structure, interest rate, latent factors, Monte Carlo Markov Chain (MCMC).

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# 1  Introduction

The term structure interest rate determination is one of the main subjects of the financial assets management. Considering the great importance of the financial assets for the economic policies conduction, how such the purchase and sale of government bonds in the primary market, it is basic to understand structure is determined.

Papers such Litterman and Scheinkman (1991) and Dai and Singleton (2000) suggest the importance of financial factors to understand the dynamics of yield curves. Those papers consider the existence of dynamic factors which determine the risk premia evolution for diverse maturities of the term structure. Generally, such dynamic factors are represented by unobserved state variables, summarising all the relevant information that determine the yield curve movement.

However, under a macrofinancial framework, the yield curve is not determined solely by financial factors. A significant literature, such as Piazzesi (2001), Ang and Piazzesi (2003) and Diebold *et al.* (2005) suggest the importance of the macroeconomic factors for determining mainly the log term yields. Empirical evidence show that changes in macroeconomic factors, coupled with well defined economic policies rules, affect the yield curve dynamics throughout the time. Ang, Dong and Piazzesi (2005), using U.S. data, merge the two frameworks, and estimate a interest rate term structure model with a time-varying risk premia and no-arbitrage. They consider simultaneously macroeconomic variable and latent factors, besides specifying the Taylor rule determining the short term interest rate.

In Brazil, the term structure behaves inversely to the one observed in the USA, with high values for the short term interest rate, and low values for the long term interest rate. We are, then, lead to the some questions. Can we claim that ADP's results hold when the term structure is inversely inclined? What are the main differences? Such questions are important because they concern mainly developing economies, which are supposed to have a long term structure lower than the short term. ADP (2005) points out that, only one latent factor produces high forecasting errors in the short term. This paper estimates the interest

rate term structure in the same way that ADP (2005), do considering, however, three latent factors. For this, we use the Monte Carlo Markov Chain econometric method, hereinafter MCMC, with Brazilian data.

The ADP (2005) model can be considered the most comprehensive, since it develops an affine model of term structure with time-varying risk premia, and also it considers the importance of interaction between macroeconomic variables and latent factors for determining the of yield curve movements. Their model is based on Duffie and Kan (1996) and the monetary policy rule, as the Taylor Rule, is one of its arguments for determining the short-term interest rate. This type of joint short and long-term interest rate modeling should provide more information about the dynamic interaction of rates for different periods.

Only a few papers already had Brazil as focus, such as Matsumura and Moreira (2007), hereinafter MM, and Shousha (2005). These papers use only one latent factor, and maturities of 12 months at most. Besides using three latent factors, we opt to use maturities of up to 60 months, with intention to extend the information horizon. The inclusion of two latent variable, by itself, shows significant contribution to understand the yield curve dynamics, catching effect not explicit in the macroeconomic factors and not caught by only one latent factor. Moreover, these papers present, in short-term, consistent results with ADP (2005), and inverse direction results in long-term, with bigger forecast errors in longer yields. This paper, however, presents results in inverse direction to those found ADP (2005).

Thus, the inclusion of the two latent variables produces singular results, with positive variations of the curves given inflation shocks and the first and the third latent factors, and negative variations resulting of output shocks and the second latent factor.

The remainder of the paper is organized as follows. Section 2 describes the theoretical model, considering an affine term structure model and a Taylor rule. Section 3 presents the econometric method used in this paper. In Section 4, we describe the dataset. Section 5 presents the results of the estimated model. Section 6 concludes the paper.

# 2   Affine term structure model

In this section, we present the main hypotheses of the term structure estimation model: affine models. This family of models, introduced by Duffie and Kan (1996), enables one to specify a time-varying risk premia, accommodating state variables with averages and covariances also varying over time.

The term structure can be understood as the perception of agents abouth the future state of the economy, there our paper proposes to draw up a model capturing the dynamic of interest rate curves, and taking into account the short-term interest rate explained by a composition of latent factors and macroeconomic variables.

Preliminarly, the interaction between the perception of future economic growth and macroeconomic variables provides the monetary authorities with important information for their policy decisions and for the forecasts of market participants in a manner that is somewhat dynamic and recursive: policy decisions today are likely to impact the actions of market participants today, influencing the interest rate curve in the following periods, which in turn will influence the decision of the monetary authorities, and so on and so forth.

Furthermore, the use of data with many more maturities makes it possible to extract more abundant and more accurate information, because long term interest rates reflect the expectations of agents on the spot rate at the long term.

In addition, our paper introduces a non parametric estimation method based on the Monte Carlo Markov Chain. In this literature, the method, according to ADP (2005), presents better results than parametric methods, which in Brazil cannot be reasonable, given the existence of extreme values.

## 2.1   The model

Following the model developed and used by Ang, Dong and Piazzesi (2005), let us indicate the dimension vector $j \times 1$ composed of state variables, as follows:

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$$X_t = \begin{bmatrix} g_t & \pi_t & f_t^u {}^\mathsf{T} \end{bmatrix}^\mathsf{T} \tag{1}$$

where:

$g_t$ is the growth rate of GDP between the periods $t-1$ and $t$;

$\pi_t$ represents the inflation rate between the periods $t-1$ and $t$;

$f_t^u$ is a latent variables vector of the term structure of the interest rate;

$\mathsf{T}$ means transpose.

Assuming that each variable follows an order 1 autoregressive-type process VAR(1), then we have:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t \tag{2}$$

where

$\Phi$ is the coefficient matrix of VAR(1);

$\Sigma$ is the standard deviation matrix;

vector $\varepsilon_t \sim i.i.d.N(0,1)$.

Within this type of model specification, the short-term interest rate can be modeled in different ways. In a less sophisticated version and in a generalized way, we can define it as:

$$r_t = \delta_0 + \delta_1^\mathsf{T} X_t. \tag{3}$$

Substituting the last formula into the vector, as we have seen previously, we have:

$$r_t = \delta_0 + \delta_{1,g} g_t + \delta_{1,\pi} \pi_t + \delta_{1,u}^\mathsf{T} f_t^u . \tag{4}$$

where $\delta_0$ is a scalar and $\delta_1$ express a vector of dimension $j \times 1$.

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The introduction of alternative forms of the Taylor Rule into the model will immediately modify equation 4. More structured Taylor Rules impose greater complexity in determining the short-term interest rate, altering the functional form of equations 3 and 4.

In our model, we can specify the pricing kernel as being an exponential function dependent on short-term interest rates $(r_t)$, the variable risk premia over time $(\lambda_t^\intercal \lambda_t)$, and the random component $(\lambda_t^\intercal \varepsilon_{t-1})$. Thus:

$$M_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t^\intercal \lambda_t + \lambda_t^\intercal \varepsilon_{t-1}\right) \tag{5}$$

As the time-varying risk premia (given the previously expressed hypotheses), it can be determined in the following way:

$$\lambda_t = \lambda_0 + \lambda_1^\intercal X_t \tag{6}$$

where $\lambda_0$ is a scalar.

From the pricing kernel we can find the stochastic discount factor $m_{t+1}$[1] :

$$m_{t+1} = \log M_{t+1} = \Lambda_0 + \Lambda_1^\intercal X_{t+1} \tag{7}$$

where $\Lambda_0$ is a scalar and $\Lambda_1$ is a vector of dimension identical to $X_{t+1}$.

With the set of instruments developed so far, it is possible to derive the expression for the term structure of an affine model. For this, it is necessary to find an expression which defines the yield of a zero-coupon bond maturing in any period $n$. Thus, denotes $y_t^{(n)}$ as the term structure we wish to find with these characteristics, we have:

$$P_t^{(n)} = \exp\left(-n y_t^{(n)}\right) \tag{8}$$

And therefore:

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[1] The derivation of the random factor of discounting and, consequently, of the price kernels in Ang, Dong and Piazzesi (2005), or in Appendix A.

$$y_t^{(n)} = -\frac{\log P_t^{(n)}}{n} = -\frac{p_t^{(n)}}{n} \tag{9}$$

where

$P_t^{(n)}$ is the price of the asset in period $t$;

$n$ is the price of the asset in period;

$p_t^{(n)}$ is the logarithm of the asset price levels in period $t$.

In this model, the price of a bond is necessarily related with exponential function of the state variables contained therein. In algebraic terms:

$$P_t^{(n)} = \exp\left(A_n + B_n^{\mathsf{T}} X_t\right) \tag{10}$$

The equations exhibited so far, especially the algebraic specification of the vector $X_t$ and the pricing kernel in its usual form enable us to develop a related term structure model. Thus we know that:

$$y_t^{(n)} = a_n + b_n^{\mathsf{T}} X_t \tag{11}$$

where:

$a_n$ is a scalar determined by $-\frac{A_n}{n}$;

$b_n$ is a vector of dimension $j \times 1$ given by $-\frac{B_n}{n}$.[2]

## 2.2 Incorporating the Taylor Rule into the model

Since the model estimates the term structure of the Brazilian interest rates is according with the existence of a variable risk premia over time, the short-term interest rate must also align itself with those objectives. That is done by introducing the rule for determining the

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[2]This derivation is in Appendix A.

short-term interest rate, namely the Taylor Rule, which will substitute the basic rule shown in the model by the equations 3 and 4.

The Taylor Rule (1993) presented in its classic version, captures the responses of the monetary authority (in the Brazilian case, the central bank) to movements, desirable or otherwise, of relevant macroeconomic variables (usually, inflation and output). These responses are given by variations in the basic short-term interest rate.

Although they present a satisfactory description of the behavior of the term structure, the models which use latent factors do not provide sufficient information about the economic nature of interest rate shocks and therefore become less relevant for an analysis of the term structure dynamics. This information will, therefore, be provided by the rule for determining short-term interest rates highly influenced by macroeconomic factors.

According to ADP (2005), any version of the Taylor rule will be compatible with the structure of the model, provided the latter belongs to the class of related models. Several versions of the Taylor Rule exist, but we estimated on the forward-looking type of Taylor Rule.

## 2.3 Forward-Looking Taylor Rule

In the case of the Forward-Looking Taylor Rule, we assume the central bank adjusts interest rates according with the expectations of real GDP growth and inflation in the coming periods.

In the event the central bank considers only those elements within a finite horizon, we have a Forward-Looking Taylor Rule whose algebraic form is given below:

$$r_t = \gamma_0 + \gamma_{1,g} E_t \left( g_{t+1} \right) + \gamma_{1,\pi} E_t \left( \pi_{t+1} \right) + \varepsilon_t \tag{12}$$

where $r_t$ is the short-term interest rate, $E_t \left( g_{t+1} \right)$ is the expected real GDP growth at period $t+1$ (or the expected output growth in the period), $E_t \left( \pi_{t+1} \right)$ is the expected inflation at period $t+1$ and $\varepsilon_t$ is the monetary policy shock of the Forward-Looking Taylor Rule.

Rewriting the equation in its vector form, we have:

$$r_t \;=\; \gamma_0 + \begin{pmatrix} \gamma_{1,g} & \gamma_{1,\pi} & 0 \end{pmatrix} E_t\left(X_{t+1}\right) + \varepsilon_t \tag{13}$$

$$=\; \gamma_0 + \begin{pmatrix} \gamma_{1,g} & \gamma_{1,\pi} & 0 \end{pmatrix} \left(\mu + \Phi X_t\right) + \varepsilon_t$$

$$=\; \gamma_0 + \left(\gamma_{1,g}e_1 + \gamma_{1,\pi}e_2\right)^{\mathsf{T}} \mu + \left(\gamma_{1,g}e_1 + \gamma_{1,\pi}e_2\right)^{\mathsf{T}} \Phi X_t + \varepsilon_t \tag{14}$$

where $e_i$ is a vector of zeros, except in the $i^{th}$ position, where it is equal to one.

Re-writing equation 13 according to the specifications of a related structure model, we have:

$$r_t = \overline{\delta}_0 + \overline{\delta}_1^{\mathsf{T}} X_t \tag{15}$$

$$\overline{\delta}_0 = \gamma_0 + \left(\gamma_{1,g}e_1 + \gamma_{1,\pi}e_2\right)^{\mathsf{T}} \cdot \mu \tag{16}$$

$$\overline{\delta}_1 = \Phi^{\mathsf{T}} \left(\gamma_{1,g}e_1 + \gamma_{1,\pi}e_2\right) + \gamma_{1,u}e_3 \tag{17}$$

with

$$\varepsilon_t = \gamma_{1,u} f_u^t \tag{18}$$

$$\gamma_{1,u} = \delta_{1,u} \tag{19}$$

The same can be done considering an infinite horizon, that is, in the event the central bank considers the entire future path of the relevant macroeconomic variables for a rate $\beta$. Thus:

$$r_t = \gamma_0 + \gamma_{1,g}\,\widehat{g}_t + \gamma_{1,\pi}\widehat{\pi}_t + \varepsilon_t \tag{20}$$

Where:

$$\widehat{g}_t = \frac{\beta}{1-\beta}e_1^{\intercal}\left(I - \beta\Phi\right)^{-1}\mu + e_1^{\intercal}\left(I - \beta\Phi\right)^{-1}X_t \tag{21}$$

and

$$\widehat{\pi}_t = \frac{\beta}{1-\beta}e_2^{\intercal}\left(I - \beta\Phi\right)^{-1}\mu + e_2^{\intercal}\left(I - \beta\Phi\right)^{-1}X_t \tag{22}$$

# 3  Bayesian Theory – The Monte Carlo Markov Chain method

A brief description of the econometric methods is given here. All the methodology is based on that used by Duffie and Kan (1996) and ADP (2005).

Estimating the model of the interest rate term structure is done using the Bayesian technique of the Monte Carlo Markov Chain (MCMC).

Bayesian econometrics can be summarized as the systematic result of probability theory: the Bayes theorem, which the immediate relationship between the conditional probability of A given B and the conditional probability of B given A can be found. In algebraic terms:

$$P\left(A|B\right) = P\left(B|A\right)\cdot P\left(A\right)\cdot P\left(B\right)^{-1} \tag{23}$$

The distribution of A given B is the principal aim rule in econometric analysis, since it describes the beliefs of the agent in relation to A after observing the occurrence of B, given

the model. Of the exist methods for extracting a sample from a given distribution , we have used the MCMC method when the methods of available distribution, actually available distribution and distortion or transformation of samples from a standard distribution are not applicable. In this case, we can simulate a stochastic process until its results derive from stationary distribution (which the distribution we are seeking) and, based on this sample, we make the statistical inferences.

Thus:

$$P\left(\Theta|y\right) = P\left(y|\Theta\right) \cdot P\left(\Theta\right) \cdot P\left(y\right)^{-1} \tag{24}$$

where $P\left(y|\Theta\right)$ is the probability of the model, $P\left(\Theta\right)$ is the prior distribuition of the parameter and $P\left(y\right)$ is the marginal distribuition of the sample.

The MCMC is a method of sampling the target distribution by constructing a Markov chain in such manner that the stationary distribution of simulated chain is exactly the target distribution we are looking for. There are two ways of building a chain whose aim is a stationary target distribution: the Metropolis-Hastings algorithm and Gibbs sampling. The advantages and disadvantages of each one are explored in the following subsection. Let us say in advance, the algorithm used in this study will be the Metropolis-Hastings.

Therefore application of the MCMC is done by simulating a random process until its results derive from a stationary distribution (this is the distribution we are looking for) and from this sample it is possible to make statistical inferences regarding the parameters. After subsequent distributions are found, the results can be summarized by calculating the values expected and the variances of the distribution found for each parameter, as follows:

$$E\left(\theta_k|y\right) = \int \theta_k p\left(\Theta|y\right) d\theta \tag{25}$$

$$Var\left(\theta_k|y\right) = \int \theta_k^2 p\left(\Theta|y\right) d\theta - \left[E\left(\theta_k|y\right)\right]^2 \tag{26}$$

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There are many reasons for justifying the use of the MCMC method in this estimate. One of the advantages of this method is the fact that it doesn't presume a numerical maximization methodology, while the validity of the methodology employed can be verified using Markov chain convergence. So we can simulate a random process in such manner that results derive from a stationary distribution, and as a result, it is possible to use this sample to make the necessary statistical inferences.

In addition, this method allows us to consider that one or more yields have been observed with measurement errors (ADP, 2005). It is also important to consider that the Bayesian estimate method provides a subsequent distribution of a part of the time vector , which latent variables are specified, and in this way we can obtain a good estimate of the monetary effect policy shocks. In addition, it is applicable to non linear and high dimension models. Thus the use of the MCMC estimate produces better results regarding the soundness and the quality of the inference made.

## 3.1   Gibbs Sampler versus Metropolis-Hastings

As already mentioned, there are two ways in which we can find the Markov Chain, so that, by defining the a priori distribution, it is possible to find the stationary distribution associated with it. These are the Gibbs Sampling and the Metropolis-Hastings algorithm. In both cases we have to know the a posteriori probability density function to the first constant, that is,

$$P\left(\Theta|y\right) \propto P\left(y|\Theta\right) \cdot P\left(\Theta\right) \tag{27}$$

The Metropolis-Hastings algorithm uses the idea of rejection methods, so that a value is generated from an auxiliary distribution and accepted as a given probability. Thus it ensures that the convergence of the chain to the equilibrium distribution is therefore the a posteriori distribution sought.

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Using this algorithm is not conditional on knowing the distributions of the parameters, doesn't require such distributions to be of the complete conditional type. Intuitively, this algorithm "decomposes" the unknown conditional distribution into two parts: a known distribution, which generates "eligible" points, and an unknown part from which the simulation acceptable criteria. Which ensure the algorithm with a correct equilibrium distribution.

The Metropolis-Hastings algorithm significantly increases the number of applications that can be analyzed, without the conditional density of the parameters being necessarily in a closed form.

All this leads us to conclude that by using the Metropolis-Hastings algorithm, the chain can remain in the same state for many iterations (EHLERS, 2007). Therefore it is convenient to monitor the time during the iterations remain in the same state, by using the average percentage of iterations for new acceptable values. It is worth pointing out that although it is possible to choose the proposed distribution in an arbitrary manner, choosing it ensures the algorithm efficiency.

Gibbs sampling is a special case of the Metropolis-Hastings algorithm in which the parameter vector elements are updated one at a time.

In the case of Gibbs sampling, the chain always moves to a new value, so do not exist acceptance-rejection mechanism of value, as in the case of the Metropolis-Hastings algorithm. Thus the transitions from one state to another occur according to the complete conditional distributions. For this to occur, the parameters of the model must have a closed form, that is, their distributions are known.

In many situations, the generation of a sample directly from the probabilities of the parameters can be complicated and even impossible, which at times makes it impossible to apply Gibbs sampling. So that justifies the option for the Metropolis-Hastings algorithm, given the modeling of the problem using latent variables as an argument, it is not possible to generate a sample using direct probabilities of the parameters, nor can we ensure specific distributions for each parameter. In this case we have to use the Metropolis-Hastings algorithm in order to get around this problem.

# 4 Data Description

In order to estimate the model, are used macroeconomic variables series and financial series of interest rate curves. In the case of the macroeconomic variables, we use the industrial production variable as proxy for GDP growth serie. Also we use the expected GDP growth in the following month, as well as the evolution of the IPCA for the evolution of inflationary expectations and the expectation regarding the evolution of the IPCA. The industrial production and IPCA series are the responsibility of the Brazilian Institute of National Statistics and Geography [3]. The expected GDP growh series is the responsibility of the Central Bank of Brazil[4]. For the data of the term structure, we use the average PRE X DI swap rate for the 1, 2, 3, 4, 5, 6, 12, 18, 24, 36, 48 and 60 month maturities obtained from the database of the Mercantile and Futures Exchange (BM&F)[5], and are considered on a monthly basis, from January 2000 to June 2008.

# 5 Estimation Results

In this section we present the results for the estimated models.

In our estimations, we use 25,000 simulations after a burn-in sample of 5,000. We report the posterior mean and posterior standard deviation of each parameter, in parentheses. The sample period is January 2000 to June 2008, with monthly data frequency.

## 5.1 The model - Estimated Parameters

Tables 1 to 5 show a posterior distribuition of the average values of the parameters, initiating for the coefficients of the VAR, and following for the coefficients of the short term interests rate, for the parameters of the equation that determines the risk premia and for standards deviation estimated of the errors of term structure observations.

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[3]http://www.ibge.gov.br
[4]http://www.bcb.gov.br
[5]http://www.bmf.com.br

As we can see, regarding to the coefficients estimated for the VAR (in Table 1), it has an inertia in the inflation and the third latent factor. The standard deviations of such estimates are low (respectively, 0.0702 and 0.0423), so that indicates the significance of these variable in the respective equations. Such results confirm situations in recent years in our economy, with persistent inflation and periods of relative monetary contraction[6].

However, the output growth doesn't show the same behavior. The estimated coefficient, in this case, is negative, so that indicates an error in trajectory of the economic activity in the studied period. Moreover, through its a high standard deviation, that the estimation of this coefficient generates a non significant result.

Regarding to the macroeconomic variables, in the equation of output growth, we see that only coefficients related to the inflation and the second latent factor are statistically significant, estimated in -0.9612 and 0.2478 (with standard deviation in 0.1837 and 0.1342, respectively). The output growth presents negative signal, but it is not statistically significant. Also we can observe that, as the coefficient of the inflation in the equation of the economic growth is negative, variation in the inflation is positive, as a consequence, the economic growth has a reduction.

In the equation of inflation in VAR, also we notice that output growth is not statistically different of zero, showing not to be significant for the determination of this variable. However, we see, for the results of the model, the high inertia of the inflation, with estimated coefficient in 0.7104. Regarding to the latent factors, we notice the significance of first and the third latent factors, opposing the no significance of the other latent factor in the determination of the inflation.

Regarding the equations of determination of the latent variable, we observe that the macroeconomic factors are very important for forecast of the same ones. This is observed by the significance of the coefficients of economic growth and inflation. Thus, we can conclude that the macroeconomic variables add important information for the modeling of latent

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[6]Remember the importance of the latent factor representing the shocks of monetary policy in diverse forms of determination of the short term interest rate.

factors and represent an addition of efficiency for the estimation of the term structure.

Also we see that the first factor is determined not only by last values, as for the inflation and the third latent factor. The three coefficients are statistically significant, and presents positive signal. The second latent factor presents, in the VAR equation, statistical significance for all the coefficients. However, all the coefficients present negative signal. Finally, the third latent factor is determined by the two macroeconomic factors (with coefficients of 0.6157 for output growth and 0.3223 for the inflation) and by the second latent factor, beyond being determined by its proper last values. In this case, all estimated coefficients present positive signal, indicating that variations in the others variables generate impact in the third latent factor, with same signal.

We can see, as Litterman and Scheinkman (1991), that the first factor is related directly to the dynamic of the inflation in the period, as well as second factor is related to proxy of economic growth. Such conclusions evidences in the analysis of the impulse-response function.

[- insert Table 1 here -]

In Table 2, we have the estimated coefficients for the standard deviation matrix of auto regressive vector (VAR). We can notice that part of the coefficients does not present statistic significance, given that the standard deviation of the estimates is very high. This matrix, however, is essential to understand as yield curves will react to shocks in the vector state variables. This is seen in the impulse-response function analysis.

[- insert Table 2 here -]

In the equation of determination of the short term interest rate (Table 3), we observe that the coefficients are significant. The coefficient for the GDP growth is negative. Thus, when the economic growth is above of its potential level, the interest rate increases. Moreover, we see that a positive variation in the inflation also generates a positive variation in the interest

rate, evidencing the great impact of the inflation on the short term interest rate. This result confirms the current Brazilian economic policy of combat to the inflation through inflation target.

We see, also, that the variation in the inflation generates a great impact on the short term interests rate. This can be understood through the dynamic between such variables, and the short space of time that exists between the moment that occurs the variation of prices and the adjustment next to the short term interest rate. The same it is not verified between the short term interest rate and the economic growth, since the adjustment between such variable is slower.

Moreover, we impose the restriction $\delta_{1,u_i} = 1$, for $i = 1, 2, 3$. This restriction doesn't modify the estimated values in the model, since the series of latent factors can be re-scheduled of arbitrary form.


[- insert Table 3 here -]


In Table 4 we have the estimated coefficients for the risk premia. The significance of these indicates that, really, the risk premia must be considered changeable in time.

Here we see the importance of the macroeconomic variables and latent factors in the determination of the risk premia variability, since that all the coefficients are significant, also those related to output growth. This result, particularly, opposes to the results evidenced for ADP (2005), which the output growth isn't a statistical significant to explain the variation of the risk premia in time. However, they confirm the results found by Hui (2006), proving changeable macroeconomic variables and latent factors are important for determination of a risk premia variation in the time. Moreover, also we can conclude the excess return changed significantly in the time.


[- insert Table 4 here -]

Finally, in table 5 we find the estimates for standard deviations of the errors of the term

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structure (in basis point). As we see, the observation standard deviation is fairly large. Also, standards deviations of the long term interest rates are relatively bigger than those calculated for the shortest rates. For example, the observation error standard deviation of the 30 days yield is 0,12 basis point, while the observation error standard deviation of the 1800 days yield is 10 basis point.

This result does not confirm the theory defended by ADP (2005), which shows that traditional affine models often produce large observation errors of the short end of the yield curve relative to other maturities. However, since we are studying the Brazilian term structure, we know that it presents an inverted pattern. Thus, we must have opposite results: standard deviation is lower in the vertices of short term and bigger in the vertices of long term. For the 30 days yield, for example, the measurement errors are comparable to, and slightly smaller than, the estimations containing latent and macro factors. These results were also found in Hui (2006).

[- insert Table 5 here -]

The difference in the results is justified because of the three latent factors. Many papers use only one latent factor, and find bigger standard deviation in short term. When we use three latent factors, the observation error in the segment of the yield curve is lower than with only one latent factor[7].

Note that the largest observation error variance occurs at the long end of the yield curve, which indicates that treating the long rate as an observable factor may lead to large discrepancies between the true latent factor and the long rate.

## 5.2   Latent factors

The monetary policy shocks identified using no-arbitrage assumptions depend on the behavior of the latent factors. In this paper, we use three latent factors as arguments of the

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[7]Here, we use three latent variables, related to the level, slope and curvature of the estimated curves.

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model. Many papers, as Hui (2006) and ADP (2005), use only one latent factor. Although, Litterman and Scheinkman (1991) opt to the use of three latent factors, with economic interpretations of level, slope and curvature of the yield curves. An interesting question is to understand which criteria defines the amount of latent factors in a model of yield curves.

The criteria of the number of latent lactors in this model is the Principal Decomposition Analysis (PCA). The central idea of this method is to find, as base in the data of the yield curves, the implicit factors that affect all the curves (that is, the latent factors) and to calculate the participation of these in the variance of spreads of yield curves. The results of the PCA can be seen in Table 6.

[- insert Table 6 here -]

In our model, the first latent factor is responsible for 93% of the variance of yield curve spreads. The second factor totalizes 6% of the variance of curves, accumulating 99.3% of the total variance. The third latent factor contributes with 0.58% of the total variance of the curve. The three factors add 99.9% of the total variance of yield curves. The others factors are responsible for 0.1% of the variance, and they are not considered in the model. Therefore, three latent factors are considered, corroborating the results of Litterman and Scheinkman (1991).

To characterize the relation between latent factors with macro factors and yields, Tables 7 and 8 report correlations of the latent factors with various instruments. Table 7 shows that the first factor is positively correlated with inflation at 43.02% and negatively correlated with output growth at - 68.13%. The same results are shown in other latent factors: the second latent factor is positively correlated with inflation at 43.50% and negatively correlated with output growth at - 68.09%. Finally, the third latent factor is positively correlated with inflation (at 44.31%) and negatively correlated with output growth (at -67.12%).

[- insert Table 7 here -]

The correlation between first latent factor and the yield range is between 74% and 99%; and for the second latent factor, the correlation range is between 73% and 99%. Finally, the correlation between third latent factor and the yield range is between 71% and 99%.

Importantly, the correlation between the latent factors and any given yield data series is not perfect, because we are estimating the latent factors by extracting information from the entire yield curve, and not just a particular yield. Moreover, we notice that the correlation increases with maturity. Recalling the results expressed in Table 5, we see that rates of long-term present prediction errors with standard deviations higher when compared to short-term rates. Thus, there is greater degree of uncertainty in long rates, which brings the responsibility of the factors not observed in the understanding of such variations.

## 5.3   The impulse-response function

We analyze the contribution of each variable of the state vector in the dynamics of the term structure. So, we intend to see which the impact in each variable of the state vector in the dynamics of the yield curves. This is made by analysing the impulse-response function. The figures are in attached, to the end of the text.

Figures 1-A to 5-A shows the response of yields of 30, 60, 90, 360, 1080 and 1800 days to a shock in state vector variables. Figures 1-B to 5-B show the response of yields of 30, 60, 90 and 360 days to a shock in state vector variables.

[- insert Figure 4 here -]

[- insert Figure 5 here -]

Figure 1 shows the response of yields to a shock in the output growth. Figure 2 shows the reply of same yields to a shock in the inflation. The Figures 3, 4 and 5 show the reply of yields to a variation in the first, second and third latent factors.

In Figure 1, we can notice a shock in the output growth generates a negative result in the term structure, moving it down. Since, the estimated coefficient for the output growth in the determination of the short term interest rate is negative, this result is in agreement with the estimated model.

In Figure 2, we see the effect of a shock in the inflation on the yield curves is in accordance with the waited forecast for the economic theory. A shock in the inflation makes with that all the term structure dislocated for top, in a movement of opening of the curve. Based in the economic theory, an actual inflation is bigger than expected inflation in the model, so that makes the monetary authority increases the short term interests rate, inducing people recalcuted your expectations of the future interest rate. This movement makes with that all the long term interest rates modified. First of all, we notice that yiels of bigger maturity react the variations in the inflation tax of more intense form, front to the variations of the stretch short of the curve. As the forecast horizon if expands, these curves of longer maturity revert it quickly to the equilibrium and it is faster than in the shorter maturity.

Also we notice, in Figure 2, a great persistence of the inflationary shocks in the yield curves. This result is expected, observing the estimated coefficients in the VAR of the state vector and the estimated coefficients for the short term interest rate.

In Figures 3, 4 and 5, we observe the effect of shocks in the latent variable on the yield curves. The first latent factor becomes related to the change in output growth , while second factor is correlated to the inflation.

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We conclude similarly by, observing the impulse-response functions of the curves before variations in the inflation and the second latent factor. The third latent factor is understood as an interaction between the other two, showing a bigger relevance of the changeable inflation, making with that a shock in this factor, generating a positive variations in the yield curves. It is important to stand out that the high inertia of the shocks of the latent factors in the term structure can be justified by the high coefficients found in the auto-regressive vector.

In all results, we can see the greatest impacts occur in long curves, which contradicts the results found by ADP (2005). Part of this result can be explained by the behavior of interest rates in different markets. In the U.S. market, interest rates for long term are higher than interest rates for short term. Thus, considering the present value of an asset, the higher the maturity, the greater the discount on the value of it, the lower their price in present value. Thus, the impact on these values, in present value, is lower when they have relatively long maturities than on short-term assets. In Brazil, however, interest rates are behaving in a contrary pattern: the interest rates for short term are high and interest rates for long term are lower. Thus it happens the opposite: since long rates are relatively low, when we bring the value of a particular asset to the present, the discount becomes small, which is reflected in major impacts to the long-term rates.

## 5.4   Taylor Rule

The models with affine structure accomodate diverse specifications of Taylor rules. We test two types of the Taylor rules: standard Taylor rule, with proxys for inflation and economic growth in level, and Forward Looking Taylor rule, considering the expectations of inflation and economic growth.

In Table 9, we have the comparison between the stantard Taylor rule determined for the model and Ordinary Least Squares (OLS). We see the estimated coefficient for the economic growth is negative, so that is a contradictory result to that expected in view of the economic

theory. However, the papers that use brazilian data, such a type of erratic behavior of the economic activity in the determination of the short term interest rate is relatively common.

[- insert Table 9 here -]

The estimated coefficients for the inflationary behavior indicate the relevance of such variable in the determination of the monetary policy. The estimated value for the coefficient by OLS is high, if it is compared with the estimated coefficient through the econometric model, in 0.801.

Also we can see that the standard-deviation of the estimates for the model are lesser than estimated by OLS[8].

The results of the Forward Looking Taylor rule[9] expressed in Table 10 show that inflation has a great importance in the determination of $r$, in relation the estimated Taylor rule in the standard form. The estimation of our model by means of a formulation of standard Taylor rule generated a coefficient for inflation of 0.801, whereas with a Forward Looking Taylor rule, this coefficient increases to 1.0084. This type of behavior reflects the great concern of the monetary authority has in the controlling the increase of the price levels.

[- insert Table 10 here -]

Finally, we can notice the estimated coefficient for the output growth starts to be positive, opposing the joined results previously, explicit in Table 10. With that the estimated coefficient for the economic growth was of -0.165, whereas it starts to be of 0.0590.

---

[8]The standard Taylor rule estimated OLS makes use only of the short term interests rate, whereas the estimate for the model uses information contained in all the term structure.

[9]Forward Looking Taylor rule with finite horizon ($k = 1$).

# 6   Comments and Conclusions

To know the term structure and the possible relations with other economic variables is a very interesting form to understand the macrofinancial dynamics of the economy. Thus, the main objective of this paper, using all the theoretical knowledge on term structure, shape the term structure, in set with a well defined monetary politics rule. Such monetary rule is capable of generationg, in singular way, information about the behavior of the term structure. Moreover, the importance of the macroeconomic variable in the determination of the yield curves is theoretically known and to including variable macroeconomic, in way to gain our analysis for the Brazilian case.

We decide to include latent factors believing that they have information about the yield curves that are not caught by the GDP growth and inflation. Many scholars questioned the possible correlation between the macroeconomic variables and the latent factors, and the validity of its inclusion in models of this type. Here, the inclusion of these factors in addition to the macroeconomic variables was made because it has intrinsic factors to the macroeconomic variables that are not captured by the coefficients when we estimated the term structure. This confirms the significance of the latent factors in the estimation of the model. If such factors were not considered, possibly the estimation errors would be extended, and the estimation would not express of adequate form the term structure.

The estimation of the model is made by the Bayesian technique of Monte Carlo Markov Chain (MCMC). This technique allows to find the posterior distributions of the parameters of the model, since it is possible to assume a prior distribution. The great advantage of this method is to allow inferences on the studied parameters, exactly in cases which we do not obtain analytical solutions. Thus, the MCMC skirt such problem through the use of an algorithm that simulates markov chains and extracts distributions that the stationary distribution of the chain is exactly the distribution that we are looking for.

The result are especially interesting since this technique produces different results from others, when they use autoregressive vectors or kalman filter. Moreover, its possible to shape

the problem considering the existence of non observed factors in set with observed variables.

With relation to the results, we can conclude the importance of the macroeconomic variables in the determination of the brazilian term structure. The yield curve is influenced for the variation of the macroeconomic variables, by short term interest rate and of the intrinsic latent factors to the term structure. Shocks in the macroeconomic variables produce a bigger effect on the short term interest rates, and minors variations on the longer rates. However, both the types of shocks are important for the evolution of the yield curve, and basic for the analysis of forecasts of future interest rate, in short term and in long term.

# 7    Appendix

## 7.1    Stochastic discount factor - Derivation

In this appendix, we describe how to compute the stochastic discount factor.

First, we know that:

$$P_t^{(n)} = E_t\left(M_{t+1}X_{t+1}\right) \tag{A1}$$

where

$P_t$ is asset price in $t$;

$M_{t+1}$ is the stochastic discount factor;

$X_{t+1}$ is the asset payoff  in $t+1$.

$$P_t^{(n)} = E_t\left(M_{t+1}P_{t+1}^{(n+1)}\right) \tag{A2}$$

Also we know that $P_t^{(n)}$ is the price of zero coupon bond of maturity $n$ at time $t$. Thus:

$$P_t^{(1)} = E_t\left(M_{t+1}\right) = \exp\left(-r_t\right) \tag{A3}$$

where $r_t$ is risk neutral rate.

We can show that $\log\left(M_{t+1}\right)$ is a linear function of the state variables. Therefore:

$$\log M_{t+1} = m_{t+1} = \Lambda_0 + \Lambda_1^\intercal X_{t+1} \tag{A4}$$

where

$\Lambda_0$ is scalar and

$\Lambda_1^\perp$ is a matrix with same dimension of $X_{t+1}$.

In equation 2 we saw:

$$X_{t+1} = \mu + \Phi X_t + \Sigma \varepsilon_{t+1} \tag{A5}$$

where

$\mu$ is a constant;

$\Phi$ is the estimated coefficient matrix of VAR(1);

$\Sigma$ is the standard deviation matrix of VAR(1);

$\varepsilon_t \sim IID\ N(0, I)$.

Substituting the equation 2 in equation A4, we have:

$$m_{t+1} = \Lambda_0 + \Lambda_1^\intercal (\mu + \Phi X_t + \Sigma \varepsilon_{t+1}) = E_t(m_{t+1}) + \Lambda_1^\intercal \Sigma \varepsilon_{t+1} \tag{A6}$$

For assumption, $M_{t+1}$ has a lognormal distribuition, thus:

$$\exp(-r_t) = \exp\left(E_t(m_{t+1}) + \frac{1}{2} var(m_{t+1})\right) \tag{A7}$$

However:

$$
\begin{aligned}
var(m_{t+1}) &= E_t\left[(m_{t+1} - E(m_{t+1}))^2\right] = \\
&= \Lambda_1^\intercal \Sigma var_t(\varepsilon_{t+1}) \Sigma^\intercal \Lambda_1 = \lambda_t^\intercal \lambda_t
\end{aligned}
\tag{A8}
$$

where

$$\lambda_t = \Sigma^\intercal \Lambda_1 = cov_t(\varepsilon_{t+1}, m_{t+1}) \tag{A9}$$

is the risk premia.

Therefore,

$$\begin{aligned} \exp\left(-r_t\right) &= E_t\left(M_{t+1}\right) = & \text{(A10)} \\ &= \exp\left(m_{t+1} - \Lambda_1^\intercal \Sigma \varepsilon_{t+1} + \frac{1}{2}\lambda_t^\intercal \lambda_t\right) \end{aligned}$$

Finally, we can conclude that:

$$m_{t+1} = -r_t - \frac{1}{2}\lambda_t^\intercal \lambda_t + \lambda_t^\intercal \varepsilon_{t+1} \qquad \text{(A11)}$$

## 7.2  Affine Model - Derivation

As we explain in section 2, we denote the $j x 1$ vector of state variables as:

$$X_t = \begin{bmatrix} g_t & \pi_t & f_t^{u\ \top} \end{bmatrix}^\top \tag{B1}$$

where

$g_t$ is the output growth from $t - 1$ and $t$;

$\pi_t$ is the inflation rate from $t - 1$ and $t$;

$f_t^u$ is a vector latente state variable that.

We specify that $X_t$ follows a VAR(1), that is:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t \tag{B2}$$

where

$\mu$ is a constant;

$\Phi$ is the estimated coefficient matrix of VAR(1);

$\Sigma$ is the standard deviation matrix of VAR(1);

$\varepsilon_t \sim IID\ N\,(0, I)$.

The short-term interest rate can be defined, in generic form, as:

$$r_t = \delta_0 + \delta_{1,g} g_t + \delta_{1,\pi} \pi_t + \delta_{1,u} f_t^{u\ \top}\ . \tag{B3}$$

To complete the model, the price kernel is specified as:

$$M_{t+1} = \exp\left( -r_t - \frac{1}{2}\lambda_t^\perp \lambda_t + \lambda_t^\perp \varepsilon_{t-1} \right) \tag{B4}$$

where the time-varying prices of risk is:

$$\lambda_t = \lambda_0 + \lambda_1^\mathsf{T} X_t \tag{B5}$$

The pricing kernel prices all assets in the economy, which are zero coupon bonds:

$$P_t^{(n)} = E_t \left( M_{t+1} P_{t+1}^{(n-1)} \right) \tag{B6}$$

which $P_t^{(n)}$ is the price of zero coupon bond of maturity $n$ at time $t$.

We can see the price of zero coupon bond of maturity 0 is equal to face value:

$$P_{t+1}^{(0)} = 1 \tag{B7}$$

Thus,

$$P_t^{(1)} = E_t \left( M_{t+1} \right) \tag{B8}$$

But, in this type of model, the price of a bond is necessarily an affine exponential function of the state variables contained therein. In algebraic terms:

$$P_t^{(n)} = \exp \left( A_n + B_n^\mathsf{T} X_t \right) \tag{B9}$$

Considering these two forms of to see the price of a bond with maturity $n$, we see that:

$$P_t^{(1)} = E_t \left( M_{t+1} \right) = \exp \left( -r_t \right) = \exp \left( -\delta_0 - \delta_1^\mathsf{T} X_t \right) = \exp \left( A_1 + B_1^\mathsf{T} X_t \right) \tag{B10}$$

Therefore:

$$A_1 = -\delta_0 \quad \text{and} \quad B_1 = -\delta_1 \tag{B11}$$

Thus:

$$
\begin{aligned}
P_t^{(n+1)} &= E_t\left(M_{t+1}P_{t+1}^{(n)}\right) &&\text{(B12)}\\
&= E_t\left(\exp\left(-r_t - \frac{1}{2}\lambda_t^{\intercal}\lambda_t + \lambda_t^{\intercal}\varepsilon_{t+1}\right)\cdot \exp\left(A_n + B_n^{\intercal}X_{t+1}\right)\right)\\
&= \exp\left(-r_t - \frac{1}{2}\lambda_t^{\intercal}\lambda_t + A_n\right)\cdot E_t\left[\exp\left(B_n^{\intercal}\left(\mu + \Phi X_t + \Sigma\varepsilon_t\right)\right) + \lambda_t^{\intercal}\varepsilon_{t+1}\right]\\
&= \exp\left(-r_t - \frac{1}{2}\lambda_t^{\intercal}\lambda_t + A_n + B_n^{\intercal}\mu + B_n^{\intercal}\Phi X_t\right)\cdot E_t\left[\exp\left(\left(B_n^{\intercal}\Sigma + \lambda_t^{\intercal}\right)\varepsilon_{t+1}\right)\right]
\end{aligned}
$$

Considering $\varepsilon_t \sim i.i.d.N\left(0,1\right)$, then $\exp\left[\left(B_n^{\intercal}\Sigma + \lambda_t^{\intercal}\right)\varepsilon_{t+1}\right] \sim \log normal$.

$$
P_t^{(n+1)} = \exp\left(-\delta_0 - \delta_1^{\intercal}X_{tt} - \frac{1}{2}\lambda_t^{\intercal}\lambda_t + A_n + B_n^{\intercal}\mu + B_n^{\intercal}\Phi X_t\right)\cdot \exp\left[E_t\left(\left(B_n^{\intercal}\Sigma + \lambda_t^{\intercal}\right)\varepsilon_{t+1}\right)\right]
$$
$$
\text{(B13)}
$$

$$
\begin{aligned}
&+ \frac{1}{2}var_t\left[\left(B_n^{\intercal}\Sigma + \lambda_t^{\intercal}\right)\varepsilon_{t+1}\right]\\
&= \exp\left(-\delta_0 - \delta_1^{\intercal}X_{tt} - \frac{1}{2}\lambda_t^{\intercal}\lambda_t + A_n + B_n^{\intercal}\mu + B_n^{\intercal}\Phi X_t\right)\cdot \exp\left[E_t\left(\left(B_n^{\intercal}\Sigma + \lambda_t^{\intercal}\right)\varepsilon_{t+1}\right)\right]\\
&+ \exp\left[\frac{1}{2}\left(B_n^{\intercal}\Sigma + \lambda_t^{\intercal}\right)var\left(\varepsilon_{t+1}\right)\left(B_n^{\intercal}\Sigma + \lambda_t^{\intercal}\right)^{\intercal}\right]\\
&= \exp\left(-\delta_0 + A_n + B_n^{\intercal}\left(\mu + \Sigma\lambda_0\right) + \frac{1}{2}B_n^{\intercal}\Sigma\Sigma^{\intercal}B_n + \left[B_n^{\intercal}\left(\Phi + \Sigma\lambda_1\right) - \delta_1^{\intercal}\right]X_t\right)
\end{aligned}
$$

Recalling that:

1. $\exp\left[\left(\lambda_t^{\intercal} + B_n^{\intercal}\right)\cdot \varepsilon_{t+1}\right]$ has a lognormal distribuition;

2. $\varepsilon_{t+1} \sim i.i.d.N\left(0,I\right)$;

3. $B_n^{\intercal}\Sigma\lambda_t$ is a scalar and, therefore, $B_n^{\intercal}\Sigma\lambda_t = \left(B_n^{\intercal}\Sigma\lambda_t\right)^{\intercal} = \lambda_t^{\intercal}\Sigma^{\intercal}B_n$;

4. finally, the risk premia is quantified as: $\lambda_t = \lambda_0 + \lambda_1^{\intercal}X_t$.

Equaling the coefficients obtained in the last equality (equation B13) with those encountered in equation of the bond price in $P_t^{(n+1)}$, we find the following equalities, according to various recursive relations:

$$P_t^{(n+1)} = \exp\left(A_{n+1} + B_{n+1}^\mathsf{T} X_t\right) \tag{B14}$$

We have:

$$
\begin{aligned}
A_{n+1} &= A_n + B_n^\mathsf{T}\left(\mu + \Sigma\lambda_0\right) + \frac{1}{2}B_n^\mathsf{T}\Sigma\Sigma^\mathsf{T}B_n - \delta_0 \\
B_{n+1}^\mathsf{T} &= B_n^\mathsf{T}\left(\Phi + \Sigma\lambda_1\right) - \delta_1^\mathsf{T}
\end{aligned} \tag{B15}
$$

with $A_n = -\delta_0$ and $B_n = -\delta_1$.

Another relevant variable to understand the related term structure of interest rate model is the expectation of the excess holding period return (HPR) of the bonds, which take on a related form, as well as the conditional average of the state variables vector.

$$HPR_{t+1}^{(n)} = \log\left(\frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}\right) - r_t = n y_t^{(n)} - (n-1)\, y_{t+1}^{(n-1)} - r_t \tag{B16}$$

Making the proper substitutions, given the equations derived so far, and bearing in mind the conditional expectation of HPR in relation to time, we find:

$$E_t\left(HPR_{t+1}^{(n)}\right) = \frac{1}{2}B_{n-1}^\mathsf{T}\Sigma\Sigma^\mathsf{T}B_{n-1} - B_{n-1}^\mathsf{T}\Sigma\lambda_0 - B_{n-1}^\mathsf{T}\Sigma\lambda_1 X_t = A_n^{HPR} + B_n^{HPR\mathsf{T}} X_t \tag{B17}$$

So it is possible to differentiate three distinct components for determining the expected excess return:

1. Jensen's inequality term: $\frac{1}{2}B_{n-1}^{\mathsf{T}}\Sigma\Sigma^{\mathsf{T}}B_{n-1}$;

2. constant risk premia (given the vector $\lambda_0$): $B_{n-1}^{\mathsf{T}}\Sigma\lambda_0$

3. variable risk premia, in keeping with the presence of the matrix $\lambda_1$ : $B_{n-1}^{\mathsf{T}}\Sigma\lambda_1 X_t$.

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## 8.1   Tables

Table 1 - VAR Dynamics - Estimated coefficients of $\Phi$

Table 1 - VAR Dynamics : $\overline{X_t} = \Phi\overline{X_{t-1}} + \Sigma\varepsilon_t$

| | | | $\Phi$ | | |
|---|---|---|---|---|---|
| | $g$ | $\pi$ | $f_1$ | $f_2$ | $f_3$ |
| $g$ | $-0.0882$ (0.0851) | $-0.9612$ (0.1837) | $0.0509$ (0.1139) | $0.2478$ (0.1342) | $-0.0672$ (0.1367) |
| $\pi$ | $-0.0019$ (0.1273) | $0.7104$ (0.0702) | $0.1928$ (0.0337) | $0.2156$ (0.0673) | $0.0311$ (0.0535) |
| $f_1$ | $-0.4917$ (0.3225) | $0.8367$ (0.0767) | $0.4500$ (0.0278) | $-0.1239$ (0.3475) | $0.4592$ (0.0850) |
| $f_2$ | $-0.1893$ (0.0514) | $-0.5369$ (0.1047) | $-0.2118$ (0.0895) | $-0.4632$ (0.0843) | $-0.5107$ (0.1163) |
| $f_3$ | $0.6157$ (0.0965) | $0.3223$ (0.0410) | $0.2048$ (0.1504) | $0.2114$ (0.0969) | $0.8448$ (0.0423) |

Table 2 - VAR Dynamics - Estimated coefficients of $\Sigma$

Table 2 - VAR Dynamics : $\overline{X_t} = \Phi\overline{X_{t-1}} + \Sigma\varepsilon_t$

| | | | $\Sigma \times 100$ | | |
|---|---|---|---|---|---|
| | $g$ | $\pi$ | $f_1$ | $f_2$ | $f_3$ |
| $g$ | $0.1170$ (0.0257) | $0.1559$ (0.1058) | $0.8145$ (0.0710) | $0.1598$ (0.0963) | $-0.1478$ (0.0622) |
| $\pi$ | $0.0403$ (0.1667) | $0.1430$ (0.1645) | $0.5025$ (0.1173) | $0.4104$ (0.1061) | $-0.1430$ (0.1560) |
| $f_1$ | $0.6615$ (0.0808) | $0.8338$ (0.0962) | $0.1930$ (0.1102) | $0.8980$ (0.1274) | $0.3337$ (0.2215) |
| $f_2$ | $0.1221$ (0.2332) | $0.3522$ (0.1207) | $0.7540$ (0.0705) | $0.3181$ (0.1539) | $0.5734$ (0.0364) |
| $f_3$ | $0.1200$ (0.2294) | $0.7253$ (0.0953) | $0.1714$ (0.1349) | $0.6385$ (0.1285) | $0.1446$ (0.0632) |

Table 3 - Short term interest rate - Estimated coefficients of $\delta_1$

| Table 3: $\overline{r_t} = \delta_{1,g}\overline{g_t} + \delta_{1,\pi}\overline{\pi_t} + \delta_{1,1}f_t^1 + \delta_{1,2}f_t^2 + \delta_{1,3}f_t^3$ | | | | |
|---|---|---|---|---|
| $\delta_1$ | | | | |
| $g$ | $\pi$ | $f_1$ | $f_2$ | $f_3$ |
| $-0.165$ (0.056) | $0.801$ (0.027) | 1 | 1 | 1 |

Table 4 - Risk Premia - Estimated coefficients of $\lambda_1$

| Table 4 - Risk Premia : $\lambda_t = \lambda_1 \overline{X_t}$ | | | | | |
|---|---|---|---|---|---|
| | $\lambda_1$ | | | | |
| | $g$ | $\pi$ | $f_1$ | $f_2$ | $f_3$ |
| $g$ | 11.9792 (0.0814) | $-9.7314$ (0.1028) | 6.0436 (0.0504) | 7.5442 (0.1180) | $-10.6612$ (0.0698) |
| $\pi$ | $-3.1625$ (0.0852) | 4.0217 (0.0581) | $-0.8654$ (0.0911) | $-9.6377$ (0.2347) | 4.0259 (0.2525) |
| $f_1$ | $-14.2979$ (0.0661) | 7.7557 (0.0905) | $-7.0872$ (0.0467) | $-4.1369$ (0.1019) | $-3.6530$ (0.0718) |
| $f_2$ | $-8.0705$ (0.0870) | $-2.2346$ (0.1855) | $-4.7254$ (0.1646) | $-2.0798$ (0.0875) | 4.5230 (0.2498) |
| $f_3$ | $-5.2854$ (0.0815) | $-10.9378$ (0.0735) | $-1.9786$ (0.0820) | $-6.3019$ (0.1038) | 1.9109 (0.1422) |

Table 5 - Standard Deviation Yield Curves - Estimated standard deviation - $\sigma_\eta^{(n)}$

| Table 5 - Standard Deviation: $\overline{y_t^n} = \overline{y_t^{(n)}} + \eta_t^{(n)}$ | | | | | |
|---|---|---|---|---|---|
| $n(days)$ | 30 | 60 | 90 | 180 | 360 |
| $\sigma_\eta^{(n)}$ | 0.00125 (0.00009) | 0.00240 (0.00010) | 0.00345 (0.00054) | 0.00398 (0.00050) | 0.00978 (0.00047) |
| $n(days)$ | 540 | 720 | 1080 | 1440 | 1800 |
| $\sigma_\eta^{(n)}$ | 0.01877 (0.00236) | 0.02418 (0.00161) | 0.04385 (0.00539) | 0.04880 (0.01194) | 0.10206 (0.0079) |

Table 6 - Latent Factors - Principal Components Analysis

Table 6 - Latent Factors - Principal Components Analysis

| Factor | Eigenvalue | Variance Prop. | Cumulative Prop. |
|--------|-----------|----------------|------------------|
| 1 | 15.8398 | 0.9318 | 0.9318 |
| 2 | 1.0449 | 0.0615 | 0.9932 |
| 3 | 0.0980 | 0.0058 | 0.9990 |
| 4 | 0.0121 | 0.0007 | 0.9997 |
| 5 | 0.0031 | 0.0002 | 0.9999 |
| 6 | 0.0010 | 0.0001 | 0.9999 |
| 7 | 0.0006 | 0.0000 | 1.0000 |
| 8 | 0.0002 | 0.0000 | 1.0000 |

Table 7 - Correlations between latent factors and macro factors

Table 7 - Correlations with Macro Factors

|  | $g$ | $\pi$ |
|--------|---------|--------|
| $f_1$ | $-0.6813$ | 0.4302 |
| $f_2$ | $-0.6809$ | 0.4350 |
| $f_3$ | $-0.6712$ | 0.4431 |

Table 8 - correlations between latent factors and yields

Table 8 - Correlation with Yields

|       | n |       |       |       |       |
|-------|--------|--------|--------|--------|--------|
|       | 30 | 60 | 90 | 180 | 360 |
| $f_1$ | 0.7469 | 0.8136 | 0.8674 | 0.9314 | 0.9741 |
| $f_2$ | 0.7298 | 0.7975 | 0.8529 | 0.9203 | 0.9676 |
| $f_3$ | 0.7130 | 0.7796 | 0.8349 | 0.9037 | 0.9550 |

|       | n |       |       |       |       |
|-------|--------|--------|--------|--------|--------|
|       | 540 | 720 | 1080 | 1440 | 1800 |
| $f_1$ | 0.9885 | 0.9938 | 0.9981 | 0.9993 | 0.9995 |
| $f_2$ | 0.9845 | 0.9914 | 0.9972 | 0.9994 | 0.9999 |
| $f_3$ | 0.9750 | 0.9840 | 0.9928 | 0.9970 | 0.9990 |

Table 9 - Standard Taylor Rule - Estimated coefficients

Table 9 - Standard Taylor Rule

$$\overline{r_t} = \gamma_{1,g}\overline{g_t} + \gamma_{1,\pi}\overline{\pi_t} + \varepsilon_t$$

|         | $\gamma_{1,g}$ | $\gamma_{1,\pi}$ |
|---------|----------------|------------------|
| $OLS$   | $-0.094$ <br>(0.0786) | 0.975 <br>(0.253) |
| $Model$ | $-0.165$ <br>(0.056) | 0.801 <br>(0.027) |

Table 10 - Forward Looking Taylor Rule - Estimated coefficients

Table 10 - Forward Looking Taylor Rule

$$\overline{r_t} = \gamma_{1,g} E\left(\overline{g_{t,t+k}}\right) + \gamma_{1,\pi} E\left(\overline{\pi_{t,t+k}}\right) + \varepsilon_t$$

| | $\gamma_{1,g}$ | $\gamma_{1,\pi}$ |
|---|---|---|
| $Model$ | 0.0590 (0.0277) | 1.0084 (0.053) |

## 8.2 Figures

Figure 1- The impulse-response function - output growth shock



Monthly

— 30 days  — 60 days  — 90 days  — 360 days  — 1080 days  — 1800 days

Zoom



Monthly

— 30 days  — 60 days  — 90 days  — 360 days
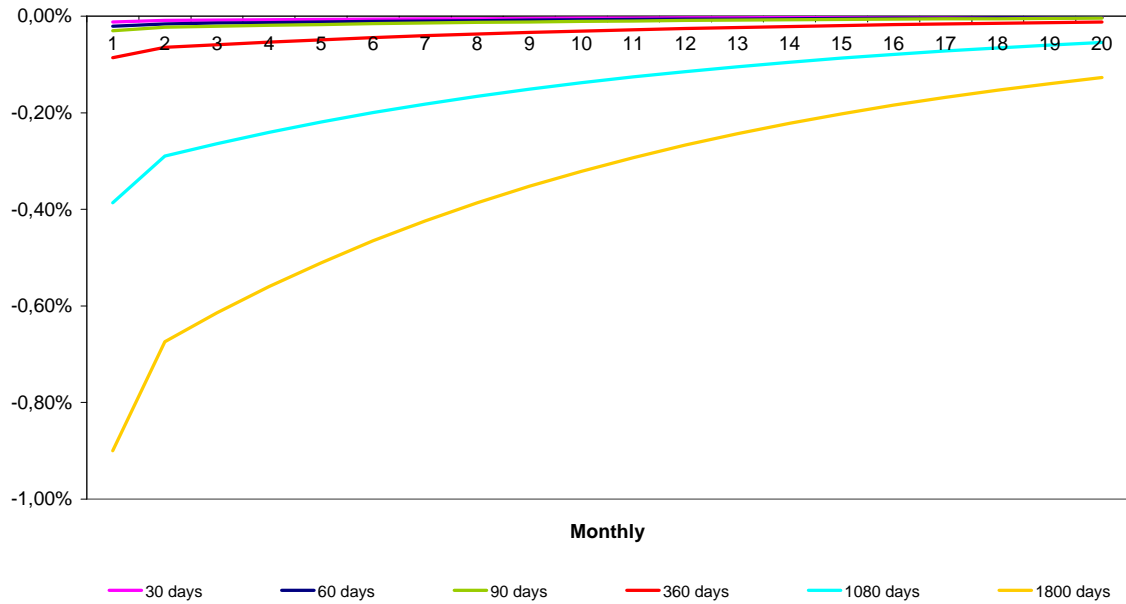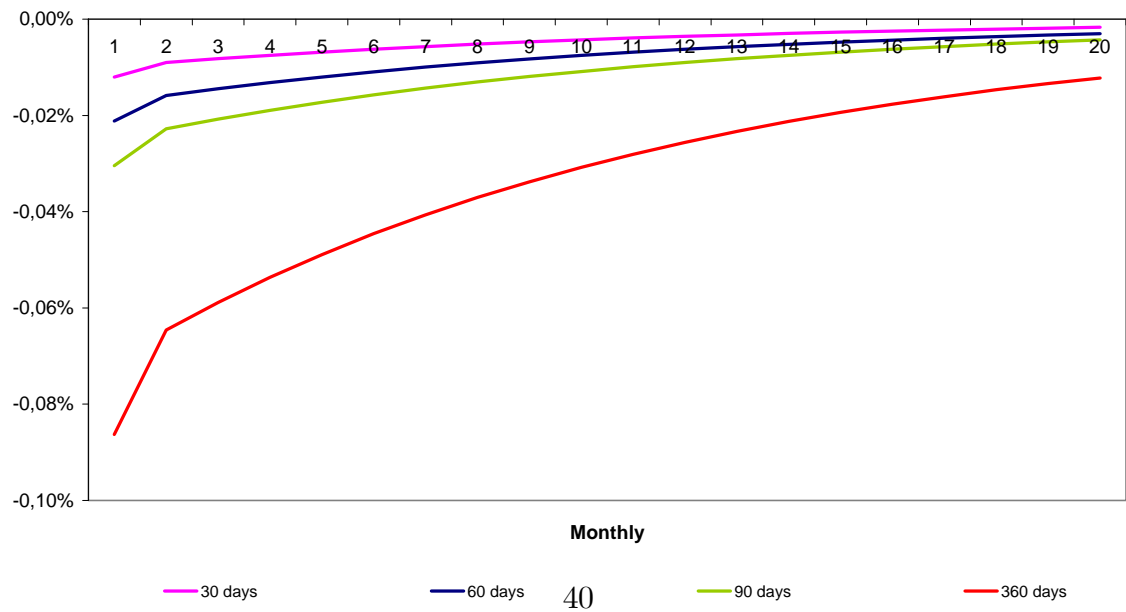
Figure 2- The impulse-response function - Inflation shock
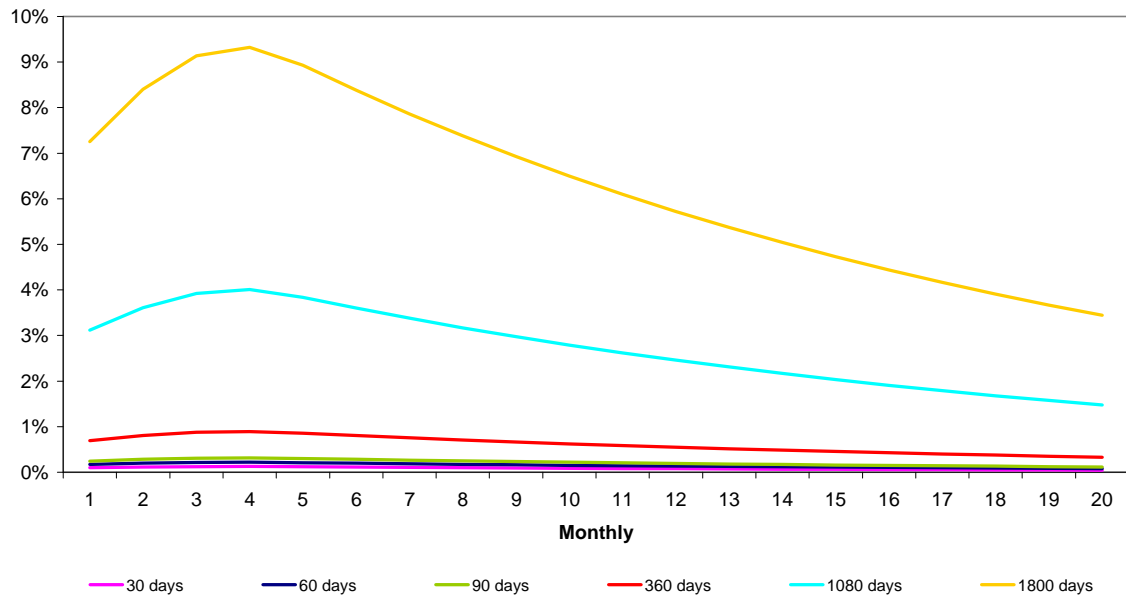


Zoom
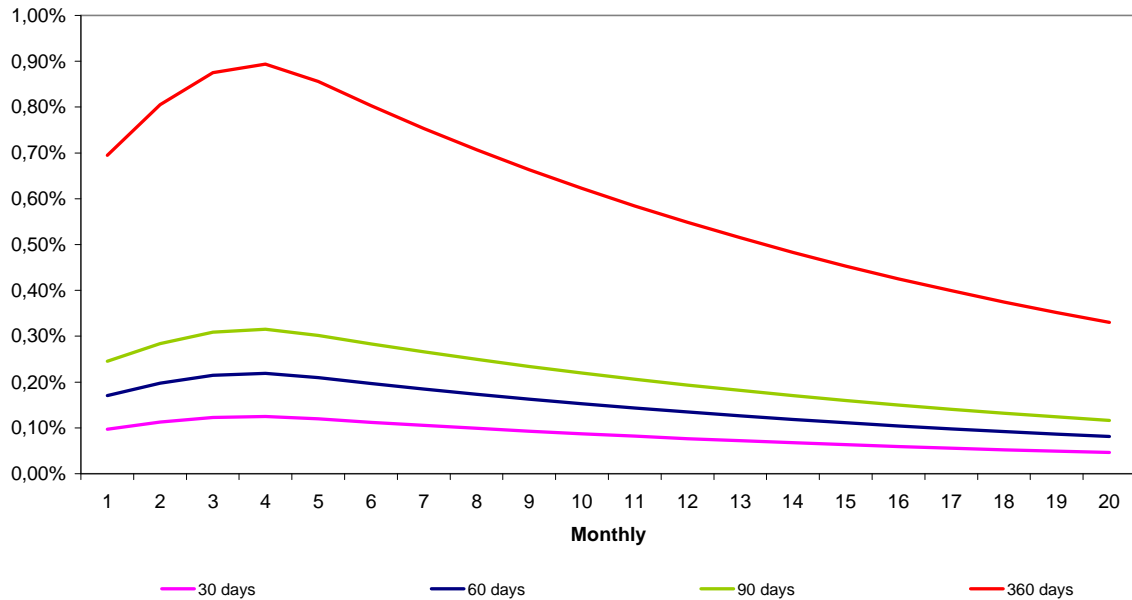
Figure 3 - The impulse-response function - First Latent Factor shock
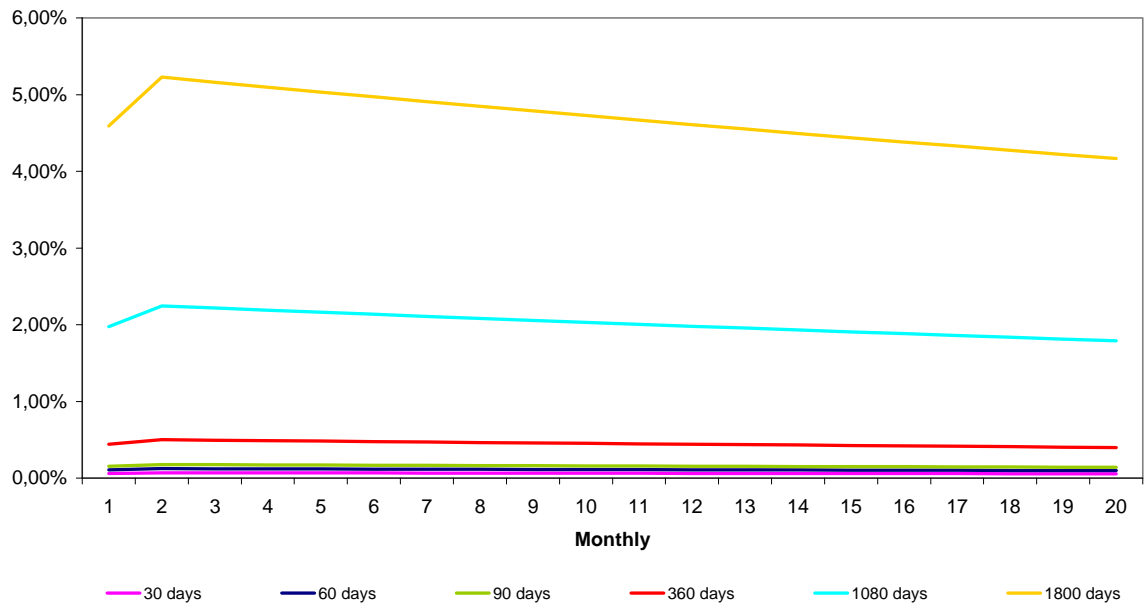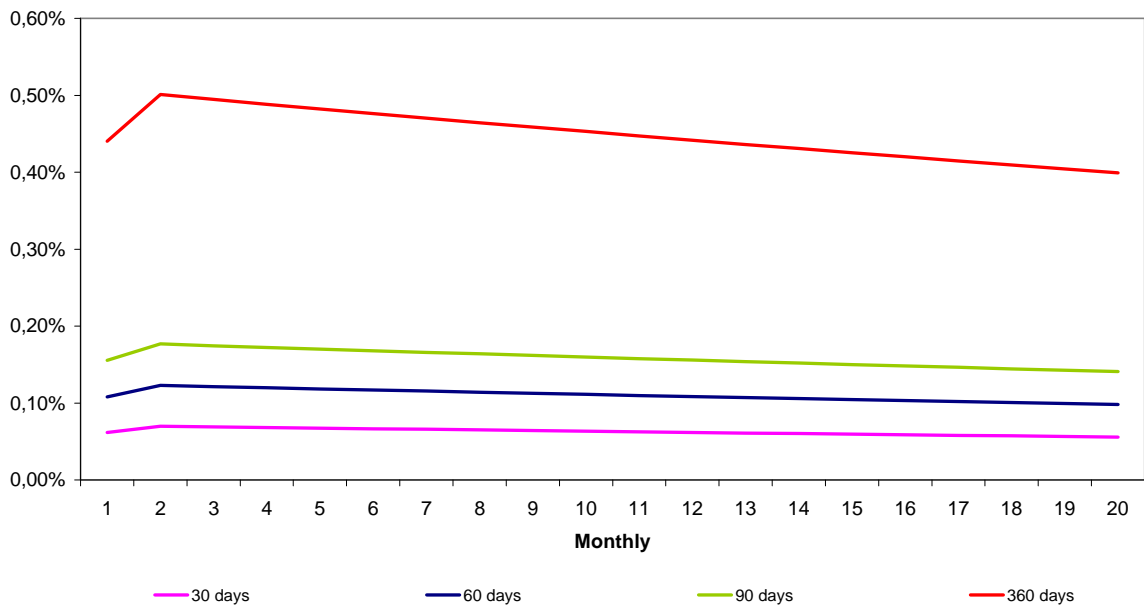
Zoom

Figure 4 - The impulse-response function - Second Latent Factor shock
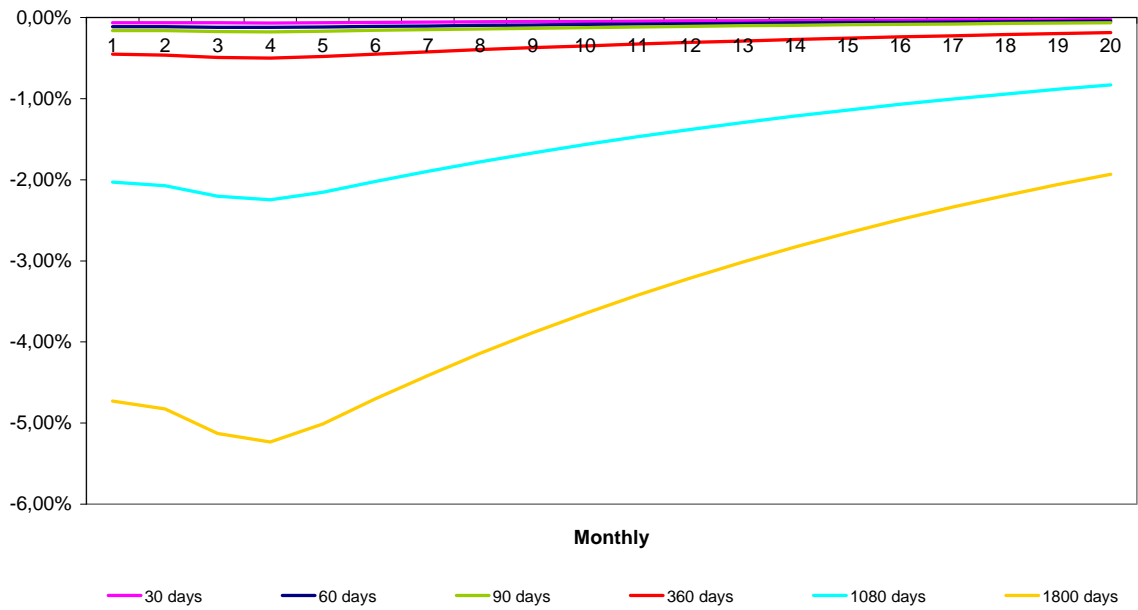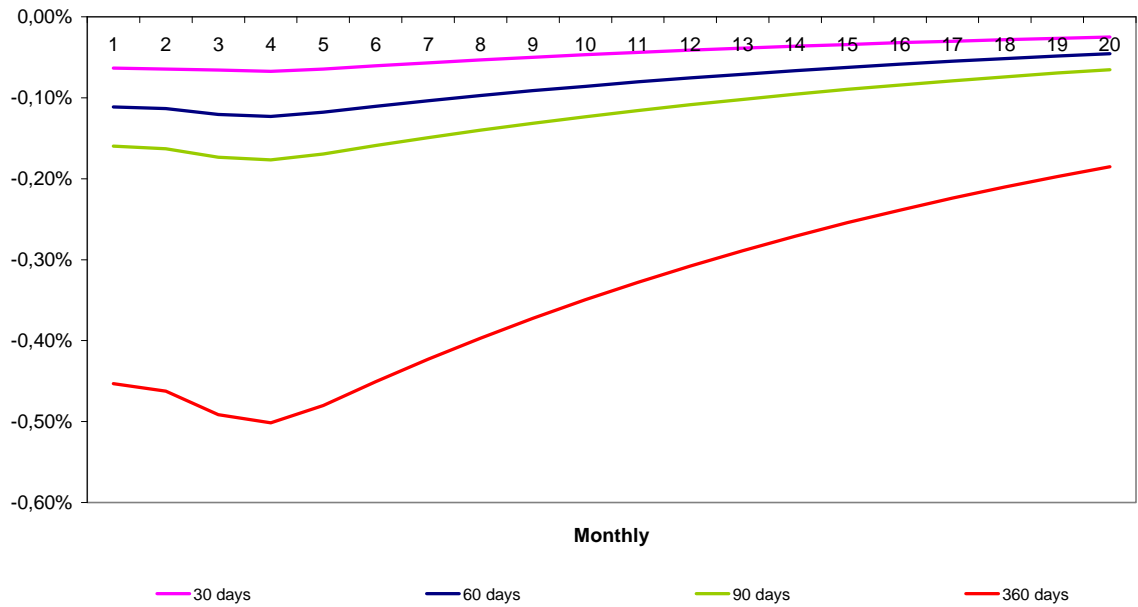


Zoom

Figure 5 - The impulse-response function -Third Latent Factor shock



Zoom

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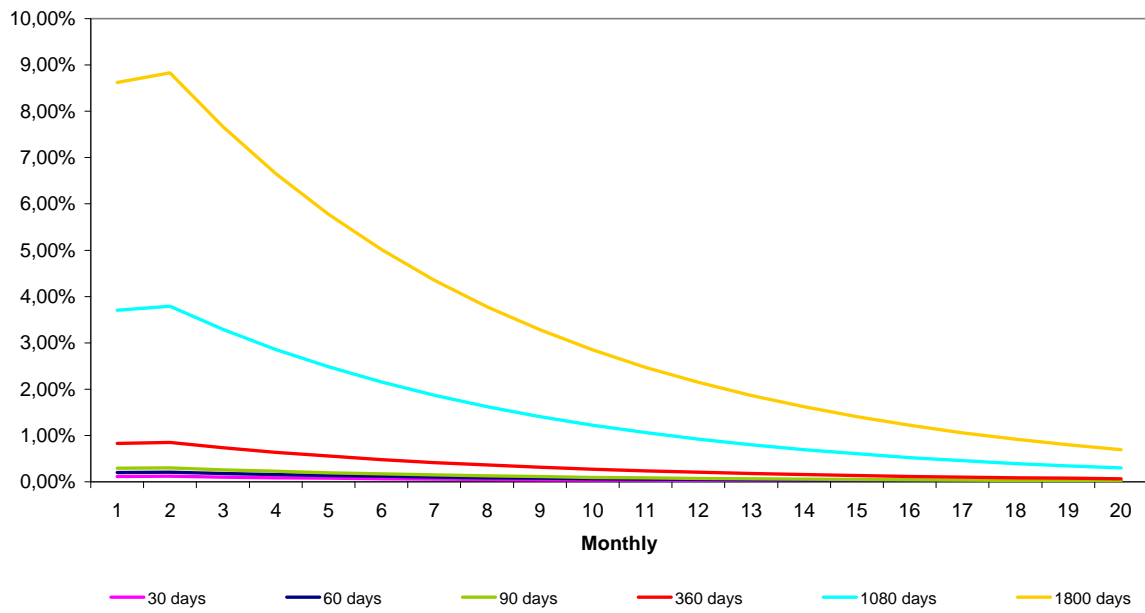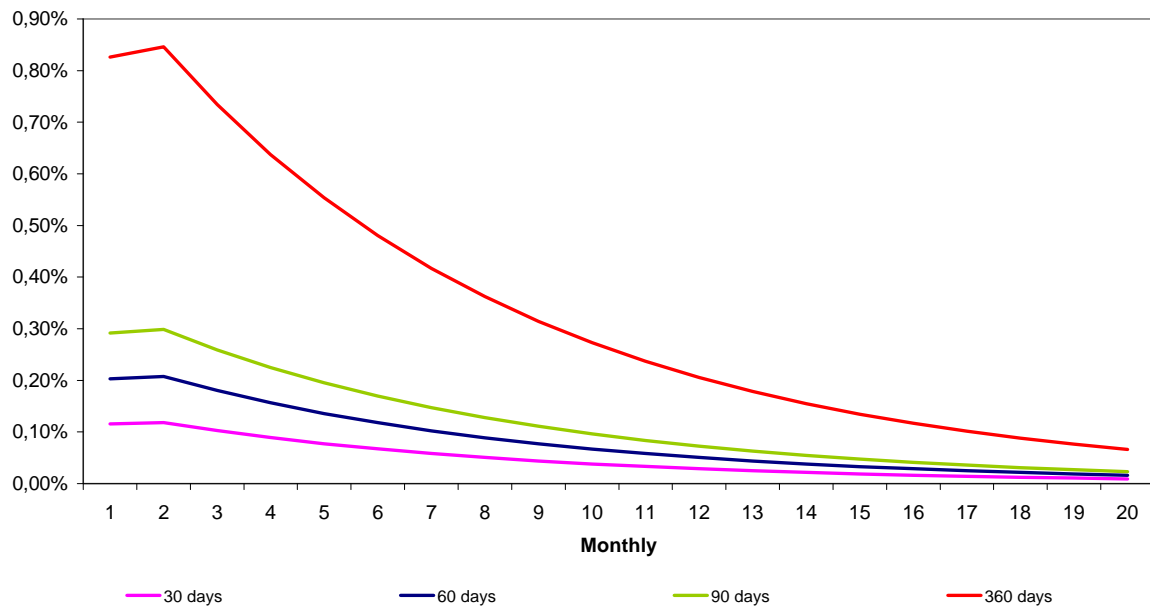
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