

Escola de Pós-Graduação em Economia – EPGE
Fundação Getúlio Vargas

Essays on International Finance, Default and Inflation

Tese submetida à Escola de Pós-Graduação em Economia da Fundação Getúlio Vargas,
como requisito para a obtenção do título de Doutor em Economia.

Aluno: Rafael Chaves Santos
Professor Orientador: Aloisio Pessoa de Araujo

Rio de Janeiro
2006

Escola de Pós-Graduação em Economia – EPGE

Livros Grátis

<http://www.livrosgratis.com.br>

Milhares de livros grátis para download.

Fundação Getúlio Vargas

Essays on International Finance, Default and Inflation

Tese submetida à Escola de Pós-Graduação em Economia da Fundação Getúlio Vargas,
como requisito para a obtenção do título de Doutor em Economia.

Aluno: Rafael Chaves Santos

Banca Examinadora:
Aloisio Pessoa de Araujo (Orientador, EPGE)
Márcia Leon (Co-orientadora, BCB)
André Minella (BCB)
Maria Cristina Terra (EPGE)
Ricardo Cavalcanti (EPGE).

Rio de Janeiro
2006

Contents

Acknowledgements	iv
Introduction	1
Chapter 1 – <i>Monetary Arrangements for Emerging Economies</i>	1
Chapter 2 – <i>Speculative Attacks, Openness and Crises</i>	23
Chapter 3 – <i>Inflation Targeting, Credibility and Coordination Aspects</i>	50

Acknowledgements

Agradeço,

Ao meu orientador, Aloísio Araujo, pela paciência e dedicação emprestada na elaboração deste trabalho. A sua orientação e generosidade manifestada em nossos sucessivos encontros tornaram possível a conclusão deste trabalho e enriqueceram a minha caminhada nestes últimos anos.

A Márcia Leon, pelo apoio incondicional em todos os momentos, e pelos ensinamentos prestados.

A todo o corpo docente da EPGE, pela contribuição fundamental para a minha formação, em especial aos componentes da banca por aceitarem dividir comigo este último momento, tão especial.

Aos funcionários da casa, pelo apoio técnico, indispensável.

Aos colegas de mestrado e doutorado, pelo companheirismo e pelos ensinamentos.

Aos meus pais e irmãos pelo apoio, amizade e conselhos.

Ao meu sogro e a minha sogra, que sempre me apoiaram, pela amizade.

A minha esposa, Ana Lucia, por ter estado ao meu lado ao longo destes últimos anos.

Ao Leonardo, meu primeiro filho, por ter colorido minha vida durante o doutorado.

Ao Mateus, por que está prestes a chegar, e me faz trabalhar de modo mais eficiente.

A Deus, por me permitir sentir a sua presença inefável que sempre me dá forças para seguir adiante, a despeito das dificuldades.

Introduction This thesis is composed by three papers, each one of them corresponding to one chapter. The first and the second chapters are essays on international finance appraising default and inflation as equilibrium outcomes for crisis time, in particular, for confidence crisis time that leads to speculative attack on the external public debt issued by emerging economies. With this background in mind, welfare effects from adopting common currency (chapter 1) and welfare effects from increasing the degree of economic openness (chapter 2) are analyzed in numerical exercises, based on DSGE framework. Cross-countries results obtained are then presented to be compared with empirical evidence and to help on understanding past policy decisions. Some policy prescriptions are also suggested. In the third chapter we look to the inflation targeting regime applied to emerging economies that are subject to adverse shocks, like the external debt crisis presented in the previous chapters. Based on a more theoretical approach, we appraise how pre commitment framework should be used to coordinate expectations when policymaker announcement has no full credibility and self fulfilling inflation may be possible.

1 Monetary Arrangements for Emerging Economies

In this paper we look at various alternatives for monetary regimes: dollarization, monetary union and local currency. We use an extension of the debt crisis model of Cole and Kehoe ([7], [8] and [9]), although we do not necessarily follow their sunspot interpretation. Our focus is to appraise the welfare of a country which is heavily dependent on international capital, due to low savings, for example, and might suffer a speculative attack on its external public debt. We study the conditions under which countries will be better off adopting each one of the regimes described above. If it belongs to a monetary union or to a local currency regime, a default may be avoided by an inflation tax on debt denominated in common or local currency, respectively. Under the former regime, the decision to inflate depends on each member country's political influence over the union's central bank, while, in the latter one, the country has full autonomy to decide about its monetary policy. The possibility that the government influences the central bank to create inflation tax for political reasons adversely affects the expected welfare for both regimes. Under dollarization, inflation is ruled out and the country that is subject to an external debt crisis has no other option than to default. Accordingly, one of our main results is that shared inflation control strengthens currencies and a common-currency regime is superior in terms of expected welfare to the local-currency one and to dollarization if external shocks that member countries receive are strongly correlated to each other. On the other hand, dollarization is dominant if the room for political inflation under the alternative regime is high. Finally, local currency is dominant if external shocks are uncorrelated and the room for political pressure is mild. We finish by comparing Brazil and Argentina recent experiences which resembles the dollarization and the local currency regimes.

1.1 Introduction

In this paper, we describe the conditions under which a country chooses one of the following monetary regimes: common currency, local currency or adopt unilaterally a strong currency as in dollarization. This country is an emerging economy whose government takes loans in the international financial market. According to the level of its external debt, the country is vulnerable to the willingness of the external creditors to keep rolling its debt. If the country is in the crisis zone and if an adverse shock hits the economy that makes the international bankers less confident that the government will pay its debt, then they suspend their credit and the government defaults.

Actually this is the description of an economy of the Cole and Kehoe ([7], [8] and [9]) model on self-fulfilling debt crisis. We use it to characterize an economy under dollarization. According to our definition, a country that dollarizes finances its public debt by issuing bonds denominated in dollars

or some other strong currency and passively follows its monetary policy.

We follow the Cole-Kehoe methodology, but do not use the interpretation of sunspot equilibrium and rely more on the possibility of a shock that affects the fundamentals of an economy. Perhaps, for a country that has a very high saving rate, the best is to drive the economy away from the crisis zone since there is some obvious bad consequences in terms of welfare related to a speculative attack. However, in an economy where the saving rate is low and therefore there is a high value to absorb foreign capital (both in terms of direct investment and also in terms of bank loans) the best is to remain in the crisis zone. Possibly, this is another reason why countries like Brazil and Argentina have been historically in the crisis zone, as shown in the history of serial default reported by Reinhart, Rogoff and Savastano ([10]). With this background in mind, such countries have to look for an optimal monetary arrangement of the type described here.

We present three alternatives for the country to choose from: in the local currency case, public debt denominated in local currency is added to the Cole-Kehoe model to describe a government that controls its monetary policy. This ability, which is absent under dollarization, consists of imposing changes in the real return on local-currency debt. The revenues collected through an inflation tax, for example, can be employed to avoid a default on the dollar denominated debt. On the other hand, the decision to inflate brings a fall in productivity. In the case of local currency we also consider the adverse possibility that the government influences its central bank to create inflation tax for political purposes in the absence of an external crisis.

Furthermore, we also describe a country in a common currency area. As in local-currency regime, an external default may be avoided by means of an inflation tax on public debt denominated in the common currency. The decision to inflate must be taken jointly. Therefore, the decision to join a monetary union depends on the correlation of external shocks among member countries and on the decision process for monetary policy. The stronger the correlation is, the higher is the possibility to use monetary policy for the purpose of smoothing shocks under common currency. The way that the decision to inflate is chosen in the monetary union also affects each member ability to smooth disturbances. A country which suffers an adverse external shock would like to have some power to pressuring the union's central bank towards inflation in order to avoid its external default. We explore two types of voting systems: either each member country can veto inflation, or has some political influence over the union's central bank. In both, the credibility of the union's monetary policy is enhanced relative to the local-currency one, since the possibility of a politically motivated central bank decreases when the decision to inflate is shared among its member countries, and not from just one.

Traditionally, the issue of an optimum currency area is based on the theoretical underpinnings developed in the 1960s by McKinnon [8], Kenen [7] and mainly Mundell [9], who is concerned with the benefits of lowering transaction costs *vis-à-vis* adjustments to asymmetrical shocks. In this paper our focus is to address financial aspects of an optimum currency area from the perspective of emerging economies. In this case, gains in credibility of the common currency relative to the loss in flexibility of monetary policy to face asymmetrical disturbances are as relevant as reduction in transaction costs. Table 1 shows the effects of flexibility in conducting the monetary policy versus currency credibility, according to the three alternative monetary regimes.

Greater autonomy in exercising monetary policy and consequently, in choosing inflation tax, has costs and benefits in terms of welfare. On one hand, welfare gains are associated to inflation as means of avoiding an external default when the indebted economy is hit by an adverse shock. On the other hand, welfare costs are related to the possibility of inflation being used as an instrument to increase public spending in the absence of shock.

One of the advantages of the Cole-Kehoe methodology is to do welfare analysis. We use their approach to evaluate the expected welfare of a member country of a monetary union. The parameters used in the model represent, in a stylized way, the Brazilian economy during the period 1998-2001. We compare the results from common currency and local-currency regimes to dollarization, in order

to appraise why Brazil and Argentina adopted different monetary arrangements between 1998 and 2001. The need to restore confidence in local currency induced both the governments to use stabilization plan that pegged their local currency to the dollar in the beginning of the 90s. However, each country were under different monetary arrangements at the end of the decade. Argentina maintained the currency-board regime, which is similar to dollarization, while Brazil adopted a floating exchange rate regime since January 1999, which resembles our local currency model. This fact led to a moderate inflation in Brazil as of 1999 and caused deflation in Argentina. The possibility of the Argentine government to obtain revenue with the devaluation of its currency would have been a valuable instrument to the financing of the international liquidity restriction caused by the Russian moratorium, occurred in August 1998. Note in Figure 1 that along with a restriction to the international credit, the current account deficit had to be reduced in both countries, but only in Brazil inflation was used to smooth the impact of the adjustment. The analysis that the absence of inflation may have worsened the Argentine crisis is aligned with Sims ([11]) who stressed advantages associated with an unexpected inflation as a means to smooth tax tightening events.

Our main result is that for a country with a highly credible currency, the local-currency regime is the best choice. Under local-currency, autonomy to inflate produces higher welfare than under common-currency, since the union's decision may be opposite to the member country choice. However, for a low credible currency, local currency is not the best choice anymore. The country will prefer dollarization or common currency depending on the correlation of external shocks.

This result refers to common currency created by emerging market economies. According to our model, the country decision of adopting the euro is more likely to the decision of adopting the dollar-currency regime. Note that the European Central Bank is uniquely independent and even if several of member countries face similar debt problems, it may be unwilling to create the requisite inflation. In the union considered here, the ability of imposing an inflation tax does give for its members an additional degree of freedom in dealing with a run on its external debt. Although it may be hard to engineer some controlled inflation tax in a country with a fairly recent history of high inflation, zero-inflation may not be desired. The recent bad experience of emerging economies with pegged exchange rate regimes has led them to search for alternative institutional framework in order to achieve currency credibility, as inflation target regime. We argue that monetary union can be another option to enhance this credibility by changing the decision process for inflation.

On a more methodological ground, the possibility that default can be welfare enhancing is in accordance with the current bankruptcy literature, which says that it is optimal to have some bankruptcy in equilibrium, contrary to conventional wisdom (see Geanakoplos, Dubey and Shubik [12], for penalties on the utility function, and Araujo, Páscoa and Torres-Martínez [1], for infinite horizon economies). Although, the risk of default should be kept under control. Accordingly, the introduction of common currency can give rise to the possibility of a better bankruptcy technology through inflation than just the repudiation of external debt, which can be quite costly.

1.2 The model with local currency

Cole and Kehoe developed a dynamic stochastic general equilibrium model in which they consider the possibility of a self-fulfilling crisis of the public debt held by international bankers occurring. We modify the original model in order to assess the welfare of an economy with two currencies and two periods. Besides the dollar currency, the local currency is added with the subterfuge that the government carries public debt in local money. The inflation tax is extracted from consumers when the government decides on the maturity date to reduce the real return of local currency bonds. This partial moratorium on local currency debt can be employed to avoid a default on the dollar denominated debt or to create inflation tax for political purposes in the absence of an external crisis. Next, we describe the economy with local currency.

1.2.1 Economic agents

The economy comprises three sectors: government, international bankers and consumers. There are infinite periods and a single good that can be consumed or saved in form of capital. Production utilizes capital and, implicitly, inelastically supplied labor.

The population of consumers is continuous and normalized to unit. Each consumer lives for infinite periods, pays a fraction ($\theta \in (0, 1)$) of his income on taxes and allocates $(1 - \theta)$ between consumption, government bonds denominated in local currency and investment so as to maximize his preferences subject to his budget constraint:

$$\begin{aligned} \max_{\{c_t, k_{t+1}, b_{t+1}\}_t} E \left[\sum_{t=0}^{\infty} \beta^t (c_t + v(g_t)) \right] \\ \text{s.t. : } c_t + k_{t+1} - k_t + q_t b_{t+1} \leq \\ [a_t \cdot f(k_t) - \delta k_t] (1 - \theta) + b_t - b_t (1 - \vartheta_t); \forall t \end{aligned}$$

c_t is private consumption and g_t is public expenditure. $v(\cdot)$ is a continuous function, differentiable, strictly concave and increasing¹. k_t is the capital stock. b_t is the government debt denominated in local currency, consisting of zero-coupon bonds maturing in one period and acquired in $(t - 1)$. ϑ_t is the government's decision variable on whether or not to inflate. When purchasing a local-currency bond, an investor pays q_t in t to receive 1 or ϕ units of good in $(t + 1)$, depending whether the government exercises inflation or not. If the government decides to inflate, then $\vartheta = \phi$; otherwise, $\vartheta = 1$. Inflation rate is given by $(\frac{1-\vartheta}{\vartheta})$, $b_t (1 - \vartheta_t)$ is the revenue that government raises by lowering the real value of its common currency debt, a_t is the productivity measure that is depreciated, if the government produces inflation or if it does not pay its dollar denominated debt. It takes one of the following values², depending on the occurrence of default or inflation previously, or in the current period:

$$\begin{aligned} (a_t | \text{inflation}) &= \alpha^\phi \\ (a_t | \text{default}) &= \alpha \\ a_t &= 1, \text{ otherwise,} \\ \text{where} &: 0 < \alpha < \alpha^\phi < 1 \end{aligned}$$

At last, $f(\cdot)$ is the production function of the economy: continuous, concave, differentiable and strictly increasing³. Each consumer is endowed with k_o units of capital and with b_o units of bonds in the initial period.

The population of bankers is also continuous and normalized to unit. Each banker is risk neutral and has an endowment \bar{x} of consumer goods in each period to be allocated between consumption x_t and government bonds denominated in dollars b_{t+1}^* . When bankers purchase a dollar-bond, he pays q_t^* units of the consumption good in t to receive 1 or 0 units of that good in $t + 1$, whether the government exercises default or not. Banker's decision to purchase government bonds is made based on his preferences as well as on his budget constraint:

$$\begin{aligned} \max_{\{b_{t+1}^*\}_t} E \sum_{t=0}^{\infty} \beta^t x_t \\ \text{s.t. : } x_t + q_t^* b_{t+1}^* \leq \bar{x} + z_t b_t^*; \forall t \end{aligned}$$

¹ $v'(0) = \infty$

²This is due to empirical data. See Simonsen and Cysne ([12]) and Cole and Kehoe ([7]).

³ $f(0) = 0$; $f'(0) = \infty$; $f'(\infty) = 0$

where z_t is the government's decision variable on whether or not to exercise default. International bankers are endowed with b_o^* units of bonds in the initial period. \bar{x} is greater enough so that the supply of credit from all international bankers meets the demand for loans.

The government is benevolent and maximizes consumers' preferences. At each period it chooses public expenditures g_t and debts of the coming periods, B_{t+1}^* and B_{t+1} . It also chooses whether it would exercise default or inflation (z, ϑ) according to the following budget constraint:

$$\begin{aligned} g_t + z_t B_t^* + \vartheta_t B_t &\leq q_t^* B_{t+1}^* + q_t B_{t+1} + \theta \cdot [a_t \cdot f(K_t) - \delta K_t] \\ z_t &\in \{0, 1\}; \vartheta_t \in \{\phi, 1\} \text{ and } \phi \in (0, 1) \\ g_t &\geq 0 \\ (z_t + \vartheta_t) &\geq 1 \end{aligned}$$

The last restriction show that it is not possible to default and to inflate at the same time.

A dollarized economy is regarded as a specific case of the economy with local currency described above. We consider that to dollarize an economy means to follow passively the monetary policy implemented by a country with a sound currency⁴. Then, to dollarize means to equalize to one the exchange rate and to zero future inflation rates.

Next we will make some simplifications so that the economy may be represented in two periods: the first one where the monetary regime is selected, the public debt is renewed and the investments decisions take place and the second, where uncertainty is solved.

Economy in Two Periods In the initial period, $t = 0$, the economy has public debt denominated in dollars, B_o^* , public debt denominated in local currency, B_o , and its productivity, a_o , is equal to one. Furthermore, there has been no shock and the public debt is renewed at the same level. The rollover cost per unit of debt, $(1 - q_o^*, 1 - q_o)$, and the investment level k_1 , depend on monetary regime previous selected.

In the next period, $t = 1$, the economy is subject to two shocks: political inflation and speculative attacks on its external debt. When uncertainty is disclosed the government chooses ϑ , a new level for its debts, decides on whether or not to exercise default and consumers choose the new level of investments. Uncertainty refers to the possibility of bankers not being willing to purchase new dollar debt from the government and to the possibility of political inflation occurs. Assuming that the government maintains constant its debt levels chosen when uncertainty is solved, and that z_t and ϑ_t remain unchanged as from $t = 1$, the economy with infinite periods can be described by only two periods, in which the second one is a perpetuity with public debt represented by a flow of interest rate over this amount.

1.2.2 Uncertainty under local currency

In the model presented here, as in the Cole-Kehoe model, the adverse shock is a restriction to foreign credit caused by a self-fulfilling debt crisis associated with a speculative attack on external public debt. The occurrence of an attack depends on a sunspot variable ζ , that is supposed distributed with uniform $[0, 1]$ and describes the bankers' (speculators) confidence that local government will not default on its external debt. This variable can be viewed as a fundamental that drives confidence and defines the equilibrium in the crisis zone: all speculators refuse to purchase new dollar bonds and default is the optimal decision or they purchase the new external debt and there is no default⁵. Next, we will introduce two additional shocks to the original model.

⁴There is no possibility of inflation.

⁵The attack may be triggered without warning in response to change in the economic fundamentals that are not explicitly described in the model, such as: change in the price of commodities that intensively take part in exports, change in the government preferences (election), reduction in international liquidity, among others.

First, it is not realistic to assume that each speculator knows in equilibrium exactly what other speculators will do. So we consider two critical values for confidence instead of one. A low value, π^d , and a high value, π^{up} . If $\zeta < \pi^d$, then the price of new dollar debt is zero, since the speculator's confidence is quite low. All of them refuse to renew their loans. Because of this, default is the optimal decision for the government whose debt is in the crisis zone. If $\zeta \geq \pi^{up}$ then all speculators are willing to purchase new external debt at a positive price and default is not optimal. But if $\pi^{up} > \zeta \geq \pi^d$ then a partial rollover takes place. In the occurrence of this moderate attack, few bankers are willing to purchase new external debt at a positive price, and so the government can renew only a fraction, φ , of its external debt. We set φ less than one but sufficient large so that government prefers to inflate than to default during moderate attack. Although we are not interested in modeling the information structure, one can think that international bankers are divided in two groups. The first and better informed one can identify three states of nature: no attack, intense attack and moderate attack. The second one can identify only two states: attack and no attack.

The second type of shock occurs when public debt is inflated away for political reasons in the absence of attacks. Political means that inflation is not an optimal decision. The probability that this shock occurs, given that there is no attack, is denoted by ψ . Therefore a political inflation shock occurs with unconditional probability equal to $\psi(1 - \pi^{up})$.

The model provides that under certain debt levels the intensity of default is proportional to the external debt crisis, that is, moderate and intense speculative attacks are respectively responded with inflation and default. Moreover, in the absence of attacks, it is optimal for the government to respect debt contracts. These critical debt levels are defined as the crisis zone in Section 4. Hereinafter we suppose that public debt is in the crisis zone.

In order to avoid creating another sunspot variable, we assume that ζ also drives consumer's actions. Accordingly, if $\zeta < \pi^d$, then consumers are sure that the government whose debt is in the crisis zone will not inflate. If $\pi^{up} > \zeta \geq \pi^d$, then they know that the government will inflate. If $\zeta \geq \pi^{up}$ neither default nor inflation is optimal and they are aware that there may be political inflation with probability ψ . Defining $\pi^{up\psi}$ as $\pi^{up} + \psi(1 - \pi^{up})$, we conclude that political shock occurs if $\pi^{up} \leq \zeta < \pi^{up\psi}$. There is no shocks with probability $(1 - \pi^{up\psi})$.

Therefore, the candidate country that elects the local-currency regime instead of the dollarization may be in one of the four possible states, s , described in Table 2.

In the beginning of period $t = 1$ uncertainty is solved with the drawing of the sunspot variable. The state s occurs if $\zeta \in \Pi_s$, where $\Pi_d \equiv [0, \pi^d]$; $\Pi_i \equiv [\pi^d, \pi^{up}]$; $\Pi_p \equiv [\pi^{up}, \pi^{up\psi}]$; and $\Pi_c \equiv [\pi^{up\psi}, 1]$. Defining $\pi^i \equiv \pi^{up} - \pi^d$, $\pi^p \equiv \pi^{up\psi} - \pi^{up}$, and $\pi^c \equiv 1 - \pi^{up\psi}$, the probability of occurrence of state s is given by π^s . All the economy sectors know the critical values and the distribution of ζ .

1.3 The model with common currency

Now, we consider a third alternative for monetary regime: a monetary union. We define a monetary union as an association of n countries plus the union's central bank. We denote each member country as member j , where $j \in \{1, 2, \dots, n\}$. When they decide to create the union their debt denominated in local currency are replaced by debt denominated in common currency. We consider that each one of them has some influence over the union's central bank, the decision-making body for inflation. The decision variable ϑ for the union will be denoted by ϑ^u , and the decision variable ϑ for each member will be denoted by ϑ^j and indicates if the member j is voting for inflation or not. Then, to join a monetary union means to share with other countries the control over inflation. We also redefine the Banker's budget constraint in order to consider the debt level from all member countries:

$$x_t + \sum_{j=1}^n q_t^{j*} b_{t+1}^{j*} \leq \bar{x} + \sum_{j=1}^n z_t^j b_t^{j*}; \forall t$$

Now, in order to estimate the welfare of country j under common currency, we must define its

influence over the union's central bank. Next, we describe two different possibilities for the decision process at the union's central bank.

In the first case, we assume that every member of the union has the right of veto over the union's decision to inflate. Then, inflation over common currency takes place only if each member votes for it. If any member prefer to default than to inflate, it votes for not to inflate. Considering the right of veto, when a country joins a monetary union, its decision to default is not changed in comparison to the local-currency regime. However, its decision to inflate may not take place if the union's decision is against it. In this case, a country has to choose between default and respect debt contracts. Note that, a dollarized economy can be regarded as a specific case of a union between an emerging market economy and a country which hypothetically always vetoes inflation.

Instead of the right of veto, we also consider an alternative voting system where each member j has some political influence on the union's central bank. In this case, when members do not agree about the decision to inflate, we assume that each member j will succeed in implementing its decision with probability pw^j . The variable pw^j is the political weight of j in the union, and the greater the value of pw^j , the greater the influence that it has on the union's central bank. Note that in this case, when a country joins a monetary union, its decision to default may be changed in comparison to the local-currency regime. If some member decide for default but the union decides for inflation, then inflation takes place. As we ruled out from the model the possibility of default and inflation at the same time, the member can not to default. Just for this case, if the public expenditure becomes negative because default is avoided, we consider that member country can default and inflate at the same time. For such situation, the productivity measure a_t turn to be $(\alpha \cdot \alpha^\phi)$.

Therefore, it is taken into account that, given a monetary union of n emerging market economies available, each economy might adopt one of the following monetary regimes: local currency, dollarization, and common currency. Under local currency, the economy does not share its currency with any country and its inflation decision is always possible to implement. Under dollarization, inflation is ruled out. Under common currency, the inflation decision is shared.

1.3.1 Uncertainty under common currency

We have already described uncertainty under local currency. Now, consider a country that elected the common-currency regime instead of the local-currency one. We assume that ζ^j has the same distribution and critical values for each member j and that all members know the correlation between events related to sunspots $\{\zeta_1^1, \dots, \zeta_1^n\}$ realization. We consider the following structure of correlation between events related to speculative attacks: the probability of occurrence of an intense attack in one country j ($Prob(\zeta^j \leq \pi^d)$) does not depend on events occurred in other members $j' \neq j$. If there is no occurrence of intense attack at the union, the events with symmetry of attacks between members are positively correlated by ρ . Thus, if ρ value is minus one and there is no occurrence of intense attack, then there is no symmetrical attacks, like "moderate attack in all members". If its value is zero the attacks occur independently and if its value is one there is no asymmetrical attacks.

Thus, if candidate countries choose to create a monetary union with the right of veto for each one of them, they will be subject to five possible states, instead of four. This happens because the voting system adds a further uncertainty to the economy. The additional state u is defined as the one where the country suffered a moderate attack but can not practice the desired inflation since at least one country voted against that. If country j votes for inflation in the absence of attack but another member vetoes its choice, then j visits state c and moves out from state p . Country j actually visits state p when decision for political inflation is aligned with the other members' vote. The probability of state d is not altered by the voting system when veto is allowed.

Table 3 sums up the five relevant events (from 16) for a member of a monetary union formed by two identical countries (A and B), as well as the probabilities of occurrence. Column s_u^A informs the state of the country A , conditional to its being part of the monetary union. The calculation of these

probabilities is detailed in the Appendix, that also presents the relevant events for a member country when the union involves three identical members and 64 possible events.

Now, if candidate countries choose to create a monetary union without the right of veto and believe that each one of them has political influence pw^j over the union's central bank, then each member j will be subject to six possible states, instead of five. This happens because the new voting system adds a further uncertainty to the economy. The new state denoted by w is defined as the one where the country suffered an intense attack, but can not practice the desired default since the union's central bank had decided for inflation. In this case, if the total tax (including inflation) is not enough to pay the external debt, we assume that this country practice default and inflation. Table 4 sums up the six relevant events (from 16) for a member of monetary union of this type and formed by two identical countries (A and B), as well as the probabilities of occurrence. The last column of Table 4 shows the probabilities of occurrence of each state, if country A prefers to maintain local currency instead of common one.

In both types of monetary union, with right of veto or without, the possibility of inflation to avoid an external default is reduced, but not ruled out as in dollarization. Inflation to avoid default is prevented by the union when s^j changes from i to u . On the other hand, political inflation in country j is also prevented when s^j changes from p to c .

1.3.2 Sequence of events

In the period $t = 0$, taking n as given, each government j chooses its monetary regime, m , among local currency, common currency and dollarization⁶. Besides, public debts is rolled over and consumers from each member choose c_0^j , b_1^j and k_1^j .

In the period $t = 1$, the events have the following order:

1. Variable ζ^j is realized, the aggregate state of economy j is $S^j = (K_1^j, B_o^{j*}, B_o^j, a_o^j = 1, \zeta^j)$ and the aggregate state of the union of n members is S , $S = \{S^1, \dots, S^n\}$.
2. Government j chooses $\vartheta^j \in \{\phi, 1\}$, taking S as given.
3. Government j , taking S , ϑ^u , and the price q_1^{j*} as given, chooses the new dollar debt $\{B_{1+\tau}^{j*}\}_{\tau>0}$.
4. International bankers, taking S , ϑ^u , and q_1^{j*} as given, choose whether to purchase $\{b_{1+\tau}^{j*}\}_{\tau>0}$ for each j .
5. Government j , taking S , ϑ^u and the price q_1^j as given, chooses the new common-currency debt $\{B_{1+\tau}^j\}_{\tau>0}$.
6. Investors from country j , considering S , q_1^j , q_1^{j*} and ϑ^u as given, choose whether to purchase common-currency bonds issued by their own country $\{b_{1+\tau}^j\}_{\tau>0}$.
7. Knowing ϑ^u , B^j , and B^{j*} , government j chooses z_1^j .
8. Consumers from country j , taking a_1^j as given, choose c_1^j and $\{k_{1+\tau}^j\}_{\tau>0}$.

1.3.3 An Equilibrium

Following Cole-Kehoe we define an equilibrium where market participants choose their actions sequentially, starting with consumers who chooses last. Consumers from each country j take as given the aggregate state S , the union's decision ϑ^u , their government's decisions $G^j \equiv (m^j, \vartheta^j, z^j, g^j, B^j, B^{j*})$, and their own decisions regarding capital k^j , and debt level b^j . In equilibrium, their choices $C^j \equiv$

⁶In the first case, j chooses $\vartheta^j \equiv \vartheta^j \in (\phi, 1)$ and n turn to be one. In the last case, j chooses $\vartheta^u = \vartheta^j = 1$.

(c^j, k^j, b^j) coincides with the aggregate capital and debt level $(., K^j, B^j)$. The consumer maximizes his utility function and chooses k_{t+1}^j that solves:

$$\frac{1}{\beta} = (1 - \theta^j) f'(k_{t+1}^j) E_t[a_{t+1}^j]$$

Furthermore consumers act competitively and are risk neutral, so they purchase public debt denominated in common currency whenever its price is equal to the expected return $1/\beta$:

$$1/\beta = E_t[\vartheta_{t+1}^u]/q_t^j$$

International bankers take as given the aggregate state S , the offer of new debt (B^j, B^{j*}) , and the debt (b^j, b^{j*}) to be received in such period. As bankers act competitively and are risk neutral too, they purchase public debt denominated in dollar from country j whenever its price is equal to the expected return at $1/\beta$:

$$1/\beta = E_t[z_{t+1}^j]/q_t^{j*}$$

Government chooses at two different times: in $t = 0$, it decides on the monetary regime m ; and in $t = 1$, after uncertainty is solved, it makes decisions at three different moments. First, in the beginning of the period, knowing the aggregate state S , it announces its vote for inflation, ϑ_s^j . After knowing union's decision, ϑ_s^u , it chooses new public debt (B_s^j, B_s^{j*}) . At last, it chooses z_s^j and g_s^j . At the beginning of period, the government is capable of anticipating capital accumulation as productivity expectation and the price that makes bankers and investors indifferent to purchasing public debt. Its optimization problem is

$$\begin{aligned} & \max_{G_{t,s}^j} E \sum_{t=0}^{\infty} \beta^t [c_{t,s}^j + v(g_{t,s}^j)] \\ \text{s.t. } & \vartheta_s^j \in \{\phi, 1\}; \forall s \in S \\ & g_{t,s}^j \leq \theta [a_{t,s}^j f(K_{t,s}^j) - \delta K_{t,s}^j] - B_{t,s}^{j*} (z_{t,s}^j - q_{t,s}^{j*}) - B_{t,s}^j (\vartheta_{t,s}^u - q_{t,s}^j); \forall s, t \\ & z_s^j \in \{0, 1\}, z_s^j + \vartheta_s^u \geq 1; \forall s, j \end{aligned}$$

Then, for each country j , an equilibrium can be defined as a list of choices $G_{t,s}^j, C_{t,s}^j, b_{t+1,s}^{j*}$, an equation of accumulation of aggregate capital $K_{t+1,s}^j$ and prices $q_{t,s}^{j*}, q_{t,s}^j$ so that, for every t, s and j :

- (i) Given $S, G_{t,s}^j, q_{t,s}^{j*}, q_{t,s}^j : C_{t,s}^j$ solve the consumer's problem.
- (ii) Given $S, C_{t,s}^j, q_{t,s}^{j*}, q_{t,s}^j : G_{t,s}^j$ solve the government problem.
- (iii) $q_{t,s}^{j*}$ and $q_{t,s}^j$ solve: $1/\beta = E_t[\vartheta_{t+1,s}^u]/q_{t,s}^j = E_t[z_{t+1,s}^j]/q_{t,s}^{j*}$.
- (iv) Given $S, B_{t+1,s}^j = b_{t+1,s}^j$.
- (v) Given $S, B_{t+1,s}^{j*} = b_{t+1,s}^{j*}$.
- (vi) Given $S, K_{t+1,s}^j = k_{t+1,s}^j$.

1.4 Debt Crisis Zone

The payoff for government j conditional to decisions z^j and ϑ^u is denoted by $U(z^j, \vartheta^u)$. The debt crisis zone is defined as the local-currency and dollar debt levels for which it is optimal for the government to respond with inflation to a moderate attack, to respond with default to an intense attack and to honor contracts in the absence of an attack. Moreover, if government debts are in the crisis zone and inflation can not be implemented during an moderate attack, then default will be the second best option. Thus, (B_o, B_o^*) will be in the debt crisis zone if the following conditions are satisfied:

$$\begin{aligned}\zeta^j &\in \Pi_d \Rightarrow U(0, 1) \geq \max \{U(1, \phi), U(1, 1)\} \\ \zeta^j &\in \Pi_i \Rightarrow U(1, \phi) \geq U(0, 1) \geq U(1, 1) \\ \zeta^j &\in \Pi_c \cup \Pi_p \Rightarrow U(1, 1) \geq \max \{U(0, 1), U(1, \phi)\}\end{aligned}$$

To construct this equilibrium, we consider the local currency debt fixed at level B_o^j for all t . The choice of parameters ϕ and φ is somewhat arbitrary but essential to obtain the crisis zone. Given φ , we can choose ϕ so that inflation is the best response only against to a moderate attack. Note that for a different moderate attack (different value of φ), the government may set a different value of ϕ in order to avoid an external default. In the numerical exercise we consider only one type of moderate attack, and thus only one value for φ .

Government's preferences also affects the crisis zone. If the government is sufficiently concerned with current public expenditures, then it would rather respond to attacks with default. Conversely, a government sufficiently concerned with private consumption would rather fully pay its debts in all states. We construct this equilibrium to obtain an intermediate and more realistic case for government preferences, where both incentives to default and to inflate are present in this crisis zone.

1.5 Numerical Exercises

In this section, we present numerical exercises where we attempt to outline under which conditions emerging economies would be better off, in terms of welfare, by joining their currencies than being on their own. We consider a monetary union between two and three countries, where each one of them can veto the union's decision to inflate. Different political influences that members have on the union's central bank are also taken into account.

The parameters used in the simulations have been chosen to portray the Brazilian economy during July 1998 to August 2001. The definition of period length is based on the Brazilian government debt whose average length varied as indicated in Table 5. The government discount factor, β , is approximated by the yearly yield on government bond issued by the US, whose values fluctuated between 4.8 and 5 percent⁷. Based on these figures, we interpret a period length as being one year and a yearly yield on risk free bonds, r , as being 0.05, which implies a discount factor β of $0.95 (= (1 + r)^{-1})$. The tax rate, θ , varied between 0.30 and 0.35 in the period and we set it equal to 0.30. The choice of the functional form was the same used by Cole and Kehoe [7], that is, $v(g) = \ln(g)$, which implies a coefficient of relative risk aversion of one. The results are very sensitive to this parameter which, besides determining the coefficient of risk aversion, defines the relative importance of public expenditure. The production function, $f(k)$, is given by $(k)^\lambda$, where capital share λ is established at 0.4. The yearly depreciation rate, δ , is equal to 0.05. The parameter α equals 0.95, assuming that default causes a permanent drop in productivity of 0.05, as in the Cole-Kehoe model. This drop is equivalent to a net present loss relative to GDP of 1.05⁸. We set φ as 0.62 and ϕ as 0.85. The correspondent inflation rate, $(1 - \phi)/\phi$, is equal to 0.18. The permanent welfare cost of inflation,

⁷Considering U.S. government bond yield. Using the U.S. discount rate reported by IMF (International Financial Statistics), the yield varies between 4.5 and 6 percent.

⁸Considering $k_{s,t} = k_o, \forall t, s$. Considering the optimal investment level the drop is equivalent to 1.7.

α^ϕ , is estimated to be 0.998. This drop is equivalent to a net present loss relative to GDP of 0.03⁹. The probability of default, π^d , and the probability of inflation, $\pi^i + \pi^p$, under the crises zone and the local-currency regime is calculated on the basis of risk premium practiced in the financial market according to the following expression:

$$\frac{1}{\beta} = (1 + r_D^{BR}) (1 - \pi^d) = (1 + r_{LC}^{BR}) (1 - (\pi^i + \pi^p) (1 - \phi))$$

where r_D^{BR} and r_{LC}^{BR} are yearly yields on Brazilian public debt denominated, respectively, in dollar and in local currency (discounting expected inflation of Brazilian currency).

Data for r_D^{BR} is available for the whole period of analysis, while r_{LC}^{BR} only since January 2002, when its value was about 0.12¹⁰. Therefore, values for π^d varied between 0.04 and 0.11 and for $(\pi^i + \pi^p)$ is evaluated at 0.42. In the simulation, π^d and π^i were fixed at 0.04, and π^p varied from 0 to 0.9. Analogous to π^p , the correlation ρ is somewhat arbitrary and varied between -0.3 and 1 in the simulation.

Second column of Table 6 sums up debt levels, investment, private consume and public expenditure used in numerical exercises. The last column also indicates the range of the actual economic variables observed in Brazil during the period 1998-2001.

1.5.1 Results

Debt Crisis Zone Following Cole and Kehoe approach, we present in Figure 2 the debt crisis zone as a function of the maturity structure of the debt for a dollarized economy subject to speculative attack (only intense, that is, $\pi^{up} = \pi^d$). Henceforth, lengthening the maturity structure means converts an initial quantity B of one-period (one year) bonds into equal quantities B_n of bonds of maturity $1, 2, \dots, N$. Then, the government redeems B_n^* bonds every period and sells B_n^* n -period bonds, where B_n^* is given by $B_n^*(1 - \beta^n) = B^*(1 - \beta)$. Results presented in Figure 2 consider the stationary participating constraint, which gives the highest debt level under which not to default is better than to default when there is no speculative attack. The no-lending condition gives the highest debt level under which not to default is better than to default when the government can not renew its old debt. For a sufficient long maturity, no-lending condition and no-stationary participation constraint coincide¹¹. In our exercise we consider the external debt as 0.45 of GDP. As the maturity gets longer, the debt required for the economy being in the crisis zone is greater. If maturity was greater than three years, for debt levels considered here, the economy would be out of the crisis zone.

Monetary Arrangements Between Two Countries with Right of Veto According to the model, the possibility to inflate depreciates the economy welfare in two ways. At first, the government may inflate even if it is not optimal to do so. Secondly, it reduces national investors' confidence in advance and, consequently, interest rate on local-currency debt rises and investment is reduced.

Our results establish conditions under which to reduce monetary autonomy is better than to maintain local-currency regime. The preferred regime depends on the risk of political inflation and on the correlation of external shocks that members are subject to. This correlation determines the likelihood of suboptimal states u , w and p occurring.

Considering a two-identical-country union, Figure 3 shows that, by changing the external shocks correlation, we obtain the optimal monetary regime for each level of the risk of political inflation, which is represent by the probability π^p . When the correlation is low and there is no risk of political inflation; investors have full confidence that the government will not inflate for political reasons

⁹In estimation of welfare cost of inflation we use Bailey's approximation and the money demand specified as kr^{-a} , where r is the logarithmic annual inflation (see Simonsen and Cysne [12]). We consider k and a equals to 0.04 and 0.6, respectively.

¹⁰Yearly yield on LTN minus inflation.

¹¹As from 45 years in our simulation.

($\pi^p = 0$); then it is better not to give up monetary autonomy. Conversely, when the risk is very high ($\pi^p = 0.9$), the economy is dollarized and monetary policy decision is transferred to the US Federal Reserve Bank. In the last case, the result is independent of correlation. Note that for high levels of political inflation the union is not desirable because it loses its inflation-inhibiting function. In such cases it is likely that both governments would vote for inflation even in the absence of attacks. For mid-risk levels, the correlation is important to define the most appropriate currency-regime. The higher the correlation, the greater the interval of risk of political inflation for which a common currency would be selected. At last, the results show how the common currency area changes in relation to the presence of an arbitrary cost associated to the creation of the union. We do this exercise, by fixing cost at one percent of GDP, only to show the sensitivity of the common currency option. In the presence of this cost, more correlation is necessary for the government to choose common currency instead of local currency.

Alternative Monetary Arrangements Figure 4 present results for a three-identical-country union and compares the results with a two-identical-country union. The addition of a new member with right of veto makes inflation less likely in the monetary union. Thus, the area of preference for a common-currency regime moves towards to higher values of external shocks correlation. This conclusion is based only on financial aspects of monetary unions and should not be taken as an optimum currency area approach since issues as international trade and factor movements are not considered in the model.

The hypothesis of identical members is convenient since it enables the conclusion that if there is an incentive to one country to join the monetary union (greater expected welfare under common-currency) there is an incentive to all, and thus the union is feasible. Relaxing such hypothesis, in the next exercises, we analyze incentives for country A to join an already established monetary union, which is defined as country B. At first, the risk of political inflation of B, π^{pB} , is fixed, while the risk of political inflation of A varies as in the previous exercises. Secondly, we consider that members have different influences over the union's decision to inflate, instead the right of veto.

Figure 5 reports results for two different values for the risk of political inflation of country B. They are fixed at $\pi^{pB} = 0.7$ and $\pi^{pB} = 0$. As expected, with the reduction of risk of political inflation in B, inflation in the union becomes less likely, and the region's preference for common currency over dollarization is increased while the region's preference for common currency over local currency is decreased. With no risk of political inflation in B, this country will vote for inflation only when it suffers a moderate attack. Therefore, country A will have less chance to inflate and common currency is less attractive for low levels of risk of political inflation. As the risk of political inflation in A and correlation rises, country A chooses common currency to improve monetary discipline.

Figure 6 shows the results when we consider that country A has some political influence over the union's central bank, instead of the right of veto. With this hypothesis, there may be inflation on common currency even if some member does not vote for it. The variable pw indicates the possibility that country A will succeed in changing the union's decision. The greater pw is, the stronger is its influence on the union's central bank. Results in Figure 6 considers π^{pB} fixed at 0.7 and pw^A as being 0, 0.4 and 0.8.

Note that over the line that separates the common-currency and the local-currency regions, welfare level is the same for both regimes. Its locus does not depend on the value of pw . Thus, if the government is indifferent about both regimes it will be indifferent about pw value.

Moreover, Figure 6 shows again that at high levels of risk of political inflation, $\pi^{pA} > 0.7$, country A looks for monetary discipline. For correlation below to 0.55, dollarization is the best monetary arrangement, because correlation is not high enough for common-currency regime to be chosen. Increasing the correlation a little (around 0.1), country A joins the monetary union as pw decreases. In this way, it attains the desired monetary discipline without having to dollarize and to discharge inflation.

In figure 7, we compare monetary regimes for member A when it can join a union where each

member has the right of veto and when it can join a union where it has no political influence over the decision for inflation. The value of π^{PB} is fixed at 0, thus in both unions there is no inflation for political reasons. In the former case, the union's central bank will inflate when both member vote for it, and in the last case when B votes. The decision of the union's central bank will depend on the decision process only when B votes for inflation but A does not¹². There are only two possibilities for this event: or $(z^A, \vartheta^A, z^B, \vartheta^B)$ is equal to $(1, 1, 1, \phi)$ or to $(0, 1, 1, \phi)$. Country B suffer a moderate speculative attack in both cases, an event with low probability of occurrence. In the first case, country A does not suffer any shock, an event with low probability of occurrence when its risk of political inflation is high. For a low level of risk of political inflation, but high level of correlation this event is rare again, due to asymmetry with the event in country B . In the second case country A suffer an intense attack, another rare event. Concluding, these two events, drawn from twelve possibilities, are very rare if we consider the region where common currency is the best option. This is the reason for, in figure 7, both pictures seems to be equal for $pw^A = 0$. In fact, if common currency is the best choice for A when veto is not allowed, then it is also the best choice when member A has the right of veto over the inflation decision.

Although these results refer to a zero-political-influence for country A , if its influence gets bigger, then the above conclusion would still be the same¹³. Monetary union with right of veto is always preferable to one with political influence decision process, because in the last one it is possible that a member decides not to inflate but the union prefers to inflate. When the union's decision prevails the forced inflation decreases the welfare or in the worst state (default with inflation), or in the best state, inflation under no-shock. Thus, according to this model, forced inflation decreases the value of common currency under political influence decision process relative to the value of common currency under the union where members have the right of veto.

Both type of union, with members having the right of veto and some political influence over inflation decision, could be described at once, with the following structure. When the union member country votes for inflation its decision is accepted by the union with probability p . When the union member country votes for no inflation, its decision is accepted by the union with probability q . If $q = 1$, we have the first type union. If $q = p < 1$ we have the second type union. We separate types descriptions for two reasons. First, for didactics purposes since in the second type additional uncertainty is considered. Second, to argue that having the right of veto (or not) is not a decisive factor to decide if adopt common currency or not, as shown in Figure 7.

Brazil and Argentina: different monetary arrangements The results obtained with the numerical exercise are aligned with the preference for dollarization by both countries in the beginning of the 90s, when to reduce inflation was the main target for monetary policy. It is also possible to appraise why different monetary regimes were adopted in Brazil and Argentina between 1998 and 2001. Brazil did not adopt a monetary arrangement similar to dollarization¹⁴, while Argentina did and suffered a default. A trivial explanation is that Argentine government erroneously thought that the adoption of foreign currency would bring economic stability, an idea largely debated in Latin America. Next, we discuss another two possible reasons for the difference in monetary arrangements: differences in risk of political inflation and differences in relative coefficient of risk aversion.

One reason for the different choice might be that the risk of political inflation of Argentina was higher than the Brazilian one. According to the results of Figure 3, the Argentinian choice would be located in the dollar region, which is characterized by higher levels of risk of political inflation relative to the local-currency one, which was the Brazilian monetary choice¹⁵. A higher risk of political

¹²Under local currency country A is subject to four possible states (c, p, i, d) and country B to three possible states (c, i, d) . Under common currency, country A will be subject to twelve possible states.

¹³Note that if we change pw^A from 0 to 0.9, the common currency area shrinks. It is also true if we increase π^{PB} .

¹⁴In the sense of the dollar-currency regime described here.

¹⁵In a very preliminary version of Araujo and Leon ([2]), written before the 2001 Argentinian crisis, the debate about

inflation can be explained by the difficulty in controlling public expenditure in Argentina where each province would have incentive to maximize the local expenditure with no commitment to sustainability of the aggregate expenditure. In Brazil, on the other hand, the institutional environment favored a little more the public expenditure control. The fiscal responsibility law completed in May 2000 is an example of political efforts towards ensuring the equilibrium of public finances.

Another reason concerns government preferences which is captured by the utility function $v(g)$. In the following exercise, we investigate different specifications for this function (for Argentina) in order to conclude how the relative coefficient of risk aversion affects the preferences for monetary arrangements. With a few exceptions, the parameters used in the simulation for the Argentine economy were the same as for Brazil. The following parameters were changed: $\theta = 0.25$, $v(g) = g^{0.01}$, $\frac{B}{GDP} = 0.5$ ¹⁶. According to the new specification for $v(g)$ the coefficient of relative risk aversion is 0.99 instead of one. With these new parameters, the government is indifferent between the local currency regime and dollarization. If the coefficient of relative risk aversion were less than 0.99, then dollarization would be preferred. If it were greater than 0.99, then local currency would be preferred. Thus, for such parameters, the region where dollarization is preferable grows along with the reduction in the risk aversion.

1.6 Conclusions

The paper brings into discussion the financial aspect about monetary regimes for countries heavily dependent on international lending and subject to political inflation. This task is accomplished by means of a macroeconomic model that incorporates microfundamentals, rational expectations and credit risk of local and foreign currency-denominated debts.

The results obtained with the numerical exercise are aligned with the preference for dollarization by economies under very high risk of political inflation. It also argued that when the risk of political inflation is moderate and external shocks correlation are high between countries, a monetary union can be an effective arrangement to increase confidence in the currency, without losing inflation as an additional instrument to smooth external shocks.

Traditionally, research on monetary union arrangements do not address the political inflation or default risk as variables of decision on adopting common currency. Such issues does not have appeal to developed economies that have strong currencies and minor risk of default. However, they are extremely relevant for emerging economies.

At last, we would like to emphasize that even though reasonable results were obtained in the numerical exercises, many aspects related to the theme were not considered, such as international trade and different types of goods.

local-currency regime versus dollarization was brought about. They argue that the local currency improves default technology and welfare is higher relative to dollarization as long as political pressure over the central bank is not too strong.

¹⁶The local currency debt was about five percent of GDP between 1998 and 2001. We consider a greater value for debt level to increase local-currency regime payoff. We also fixed the risk of political inflation at 0.53 and changed the parameter ϕ from 0.85 to 0.5. With such changes, the government is indifferent between the local currency regime and dollarization.

References

- [1] Araujo, A.; Páscoa, M. R.; Torres-Martínez, J.P.(2002):“Collateral Avoids Ponzi Schemes in Incomplete Markets”, *Econometrica*, v.70(4), 1613-1638.
- [2] Araujo, A. and Leon, M. (2000): “Speculative Attacks on Debts and Dollarization”, *Anais do XXII Encontro Brasileiro de Econometria*, December.
- [3] Cole, H.L.; Kehoe, T.J. (1996): “A Self-Fulfilling Model of Mexico’s 1994-1995 Debt Crisis”, *Journal of International Economics*, 41:309-330.
- [4] Cole, H.L.; Kehoe, T.J. (1998): “Self-Fulfilling Debt Crises”, *Research Department Staff Report 211*, Federal Reserve Bank of Minneapolis.
- [5] Cole, H.L.; Kehoe, T.J. (2000): “Self-fulfilling Debt Crises”, *Review of Economic Studies*, London: Blackwell, v. 67, n. 1, p. 91-116
- [6] Dubey, P.; Geanakoplos, J.; and Shubik, M. (2005): “Default and Punishment in General Equilibrium” *Econometrica*, 73(1).
- [7] Kenen, P.A. (1969):“The Optimum Currency Area: An Eclectic View”, R.A. Mundell and A. K. Swoboda (eds.), *Monetary Problems of the International Economy*. Chicago: University of Chicago Press.
- [8] McKinnon, R. A. (1963):“Optimum Currency Areas”,*American Economic Review*, v.53, n.4, p.717-725, September.
- [9] Mundell, R.A. (1961): “A Theory of Optimum Currency Areas”, *American Economic Review*, v.51, n.4, p. 657-665, Nov. 1961
- [10] Reinhart, C.M. ; Savastano, M.A. ; Rogoff, C.S. (2003):“Debt Intolerance”; NBER Working Paper No.9908. August.
- [11] Sims, C. (1999):“Fiscal Consequences for Mexico of Adopting the Dollar”, available in <<http://www.princeton.edu/~sims/#Papers>>.
- [12] Simonsen, M.H.; Cysne, R.P. (1994): “Welfare Cost of Inflation: The Case for Interest-Bearing Money and Empirical Estimates for Brazil”, *Ensaio Econômicos*, EPGE, Fundação Getulio Vargas, n.245, April.

1.7 Appendix

Next table presents the events at monetary union between two identical countries ($A + B$), given no occurrence of intense shock.

Event	A - Vote	Decision	State	Symmetry	Probability
(s^A, s^B)	ϑ^A	ϑ^u	$s^{A,u}$	Attacks	$Prob((s^A, s^B))$
(c, i)	1	1	c	n	$\pi_A^c \pi_B^i (1 - \rho) \mu$
(c, p)	1	1	c	y	$\pi_A^c \pi_B^p (1 + \rho) \mu$
(c, c)	1	1	c	y	$\pi_A^c \pi_B^c (1 + \rho) \mu$
(p, c)	ϕ	1	c	y	$\pi_A^p \pi_B^c (1 + \rho) \mu$
(i, c)	ϕ	1	u	n	$\pi_A^i \pi_B^c (1 - \rho) \mu$
(i, p)	ϕ	ϕ	i	n	$\pi_A^i \pi_B^p (1 - \rho) \mu$
(i, i)	ϕ	ϕ	i	y	$\pi_A^i \pi_B^i (1 + \rho) \mu$
(p, p)	ϕ	ϕ	p	y	$\pi_A^p \pi_B^p (1 + \rho) \mu$
(p, i)	ϕ	ϕ	p	n	$\pi_A^p \pi_B^i (1 - \rho) \mu$

Where

$$\left(\begin{matrix} P_{NS} \\ P_S \end{matrix} \right) \equiv \left(\begin{matrix} \pi_A^c \pi_B^i + \pi_A^i \pi_B^c + \pi_A^p \pi_B^p + \pi_A^p \pi_B^i \\ \pi_A^c \pi_B^p + \pi_A^c \pi_B^c + \pi_A^i \pi_B^i + \pi_A^p \pi_B^p + \pi_A^p \pi_B^c \end{matrix} \right), \text{ and } \mu \equiv \frac{P_{NS} + P_S}{P_{NS} + P_S + \rho(P_S - P_{NS})}.$$

$$\text{if } \rho \in [-1, 1] \Rightarrow P_{NS} + P_S + \rho(P_S - P_{NS}) \geq 0 \Rightarrow \mu \geq 0.$$

$$P_{NS} + P_S = P_{NS} (1 - \rho) \mu + P_S (1 + \rho) \mu$$

$$P_{NS} + P_S = \pi_A^c (\pi_B^i + \pi_B^c + \pi_B^p) + \pi_A^i (\pi_B^c + \pi_B^i + \pi_B^p) + \pi_A^p (\pi_B^i + \pi_B^p + \pi_B^c)$$

$$P_{NS} + P_S = (\pi_B^i + \pi_B^c + \pi_B^p) (\pi_A^c + \pi_A^i + \pi_A^p)$$

$$P_{NS} + P_S = (1 - \pi^d) (1 - \pi^d) = (1 - \pi^d)^2$$

$$P_S - P_{NS} = P_{NS} + P_S - 2P_{NS} = (1 - \pi^d)^2 - 2P_{NS}$$

$$P_S - P_{NS} = (1 - \pi^d)^2 - 2\pi_B^i (\pi_A^c + \pi_A^p) - 2\pi_A^i (\pi_B^c + \pi_B^p)$$

$$P_S - P_{NS} = (1 - \pi^d)^2 - 2\pi_B^i (1 - \pi^d - \pi_A^i) - 2\pi_A^i (1 - \pi^d - \pi_B^i)$$

$$\pi_A^i = \pi_B^i = \pi^i \Rightarrow P_S - P_{NS} = (1 - \pi^d)^2 - 4\pi^i (1 - \pi^d) + (2\pi^i)^2$$

$$P_S - P_{NS} = [(1 - \pi^d) - 2\pi^i]^2 > 0$$

$$\mu = \frac{(1 - \pi^d)^2}{(1 - \pi^d)^2 + \rho[(1 - \pi^d) - 2\pi^i]^2}.$$

If ρ value is -1 and there is no occurrence of intense attack then there is no occurrence of symmetrical attack. If its value is 0 the shocks occur independently and if its value is 1 there is no occurrence of asymmetrical moderate attack. Table 3 sums up the five relevant events (from 16) for a member A and $n = 2$. If $n = 3$ we have:

s_u^A	Probability
d	π^d
c	$\pi^c (\pi^d)^2 + 2\pi^c \pi^d \pi^i + 4\pi^c \pi^p \pi^d + 2\pi^d (\pi^c)^2 + 2\pi^d (\pi^p)^2 + 2\pi^p \pi^d \pi^i + \pi^p (\pi^d)^2 + \dots$
	$\dots (1 - \rho) \mu [\pi^c (\pi^i)^2 + 2\pi^i (\pi^c)^2 + 4\pi^c \pi^p \pi^i] + (1 + \rho) \mu [3\pi^c (\pi^p)^2 + 3\pi^p (\pi^c)^2 + (\pi^c)^3]$
u	$2\pi^c \pi^d \pi^i + \pi^i (\pi^d)^2 + 2\pi^p \pi^d \pi^i + 2\pi^d (\pi^i)^2 + (1 - \rho) \mu [2\pi^c (\pi^i)^2 + \pi^i (\pi^c)^2 + 2\pi^c \pi^p \pi^i]$
i	$(1 - \rho) \mu [2\pi^p (\pi^i)^2 + \pi^i (\pi^p)^2] + (1 + \rho) \mu (\pi^i)^3$
p	$(1 - \rho) \mu [\pi^p (\pi^i)^2 + 2\pi^i (\pi^p)^2] + (1 + \rho) \mu (\pi^p)^3$

$$\text{Where, } \left(\begin{matrix} P_{NS} \\ P_S \end{matrix} \right) = \left(\begin{matrix} 3\pi^c (\pi^i)^2 + 6\pi^c \pi^p \pi^i + 3\pi^i (\pi^c)^2 + 3\pi^p (\pi^i)^2 + 3\pi^i (\pi^p)^2 \\ (\pi^p)^3 + (\pi^c)^3 + (\pi^i)^3 + 3\pi^c (\pi^p)^2 + 3\pi^p (\pi^c)^2 \end{matrix} \right).$$

Tables and Figures

Table 1: Monetary Regimes Trade-offs

Regime	Flexibility	Credibility
Local Currency	total	low
Common Currency	partial	medium
Dollarization	null	high

Table 2: States Under Local Currency in the Crisis Zone

States	Shocks	Actions
c	none	respect contracts
p	political inflation	inflation
i	moderate attack	inflation
d	intense attack	default

Table 3: Monetary Union Between Members With Right of Veto (n=2)

s_u^A	Probability
d	π^d
c	$\pi^d(\pi^c + \pi^p) + \pi^c(1 + \rho)\mu[2\pi^p + \pi^c] + (1 - \rho)\mu[\pi^c\pi^i]$
u	$\pi^i(\pi^d + \pi^c(1 - \rho)\mu)$
i	$\pi^i\mu(\pi^p(1 - \rho) + \pi^i(1 + \rho))$
p	$\pi^p\mu(\pi^p(1 + \rho) + \pi^i(1 - \rho))$
where $\mu = \frac{(1 - \pi^d)^2}{(1 - \pi^d)^2 + \rho(1 - \pi^d - 2\pi^i)^2}$.	

Table 4: Monetary Union Between Members With Political Influence

s^A	Probability Under Common Currency (n=2)	(n=1)
d	$pw^A \cdot \pi^d + (1 - pw^A) \cdot [\pi^d \pi^d + \pi^d \pi^c]$	π^d
c	$pw^A \cdot \pi^c + (1 - pw^A) \cdot \{\pi^d(\pi^c + \pi^p) + \pi^c(1 + \rho)\mu[\pi^p + \pi^c]\}$	π^c
u	$pw^A \cdot 0 + (1 - pw^A) \cdot [\pi^i(\pi^d + \pi^c(1 - \rho)\mu)]$	0
w	$pw^A \cdot 0 + (1 - pw^A) \cdot [\pi^i \pi^d + \pi^p \pi^d]$	0
i	$pw^A \cdot \pi^i + (1 - pw^A) \cdot [\pi^i \mu(\pi^p(1 - \rho) + \pi^i(1 + \rho))]$	π^i
p	$pw^A \cdot \pi^p + (1 - pw^A) \cdot [\mu([\pi^p \pi^p + \pi^c \pi^p](1 + \rho) + (1 - \rho)[\pi^p \pi^i + \pi^c \pi^i])]$	π^p

Table 5: Brazilian Public Debt Length (Years)

Length	Model	Brazil (98-01)
Average Maturity	1	[0.4 , 2.2]
Average Duration	1	[0.2 , 0.9]

Table 6: Economy in the Crisis Zone

Variables relative to GDP	Model ($t = 0$)	Brazil (98-01)
External debt	$\frac{(B^*)}{f(K)} = 45$	[31 , 45]
External public debt	$\frac{(B^*)}{f(K)} = 45$	[9 , 24]
Local currency public debt	$\frac{(B)}{f(K)} = 30$	[27 , 31]
Capital outflow	$\frac{B^*(1-q^*)}{f(K)} = 4$	-
Investment	$\frac{\delta K}{f(K)} = 16$	[20 , 22]
Private consumption	$\frac{c}{f(K)} = 60$	[61 , 62]
Public expenditure	$\frac{g}{f(K)} = 20$	[19 , 19]

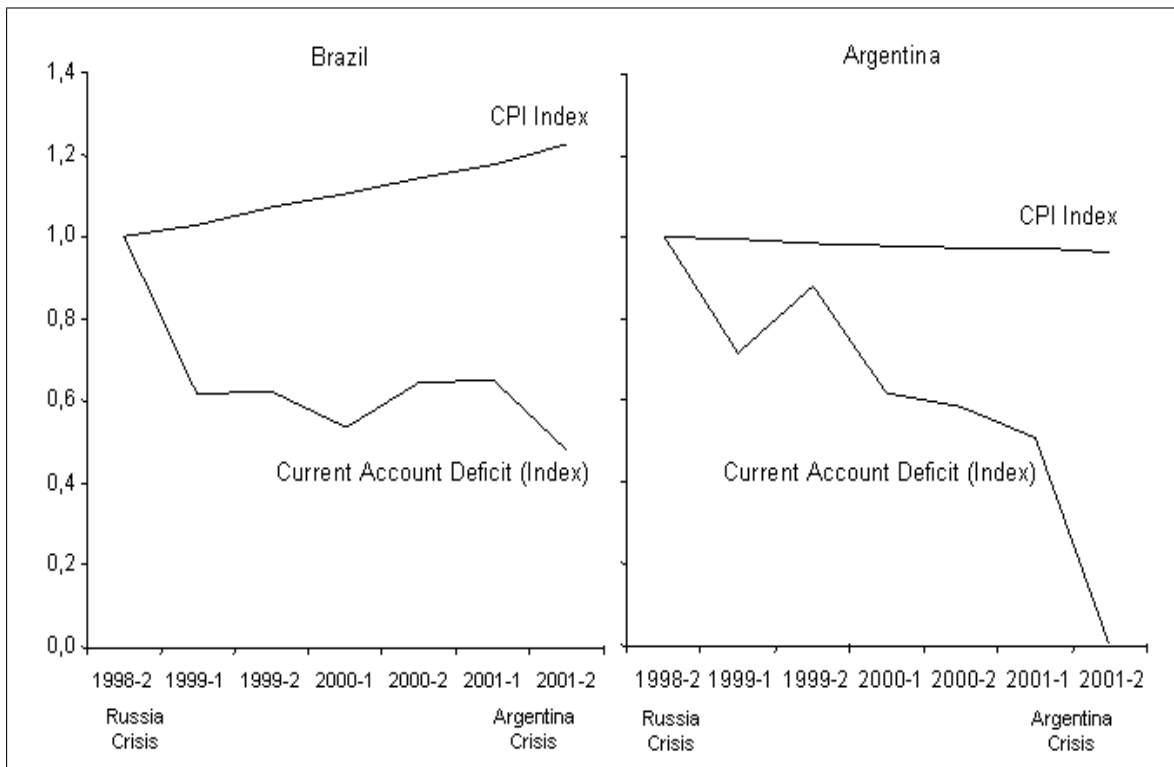


Figure 1: Inflation versus Current Account Adjustment

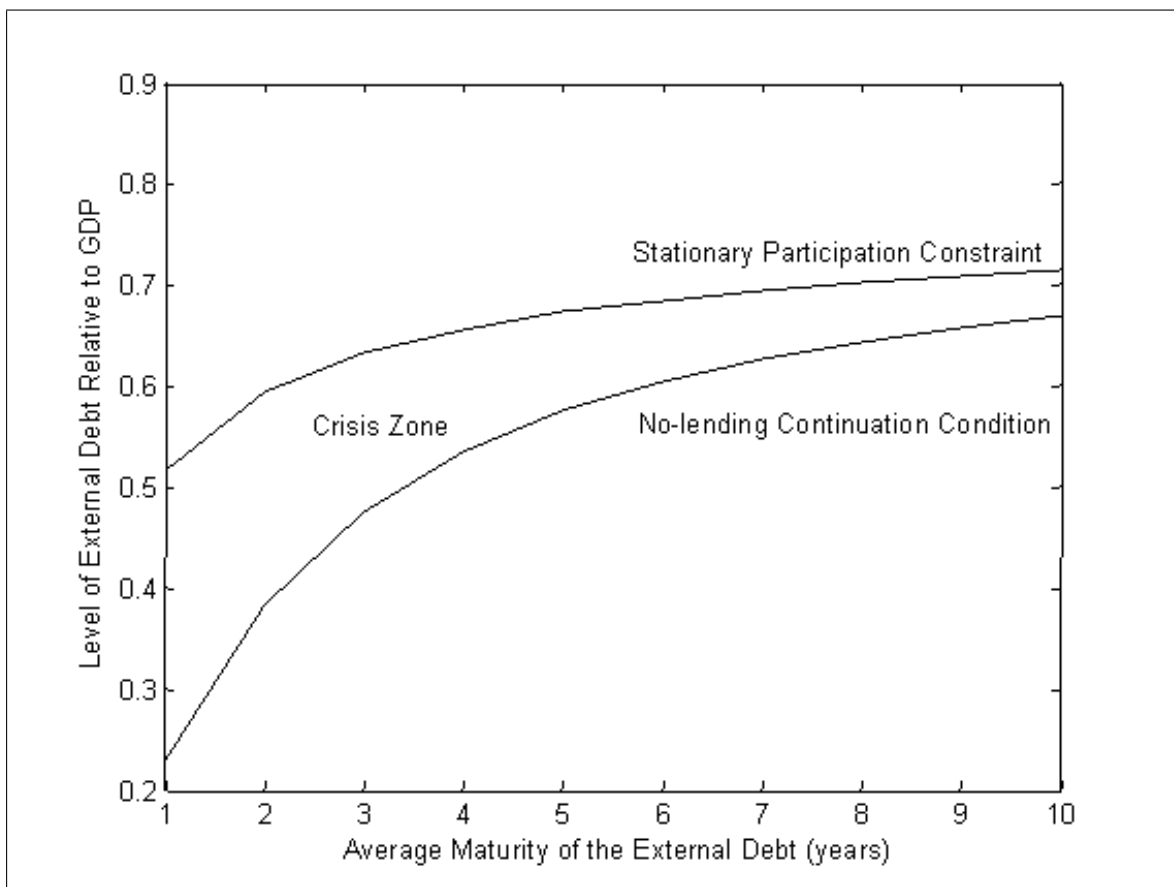


Figure 2: Debt Crisis Zone and Average Maturity of the External Debt

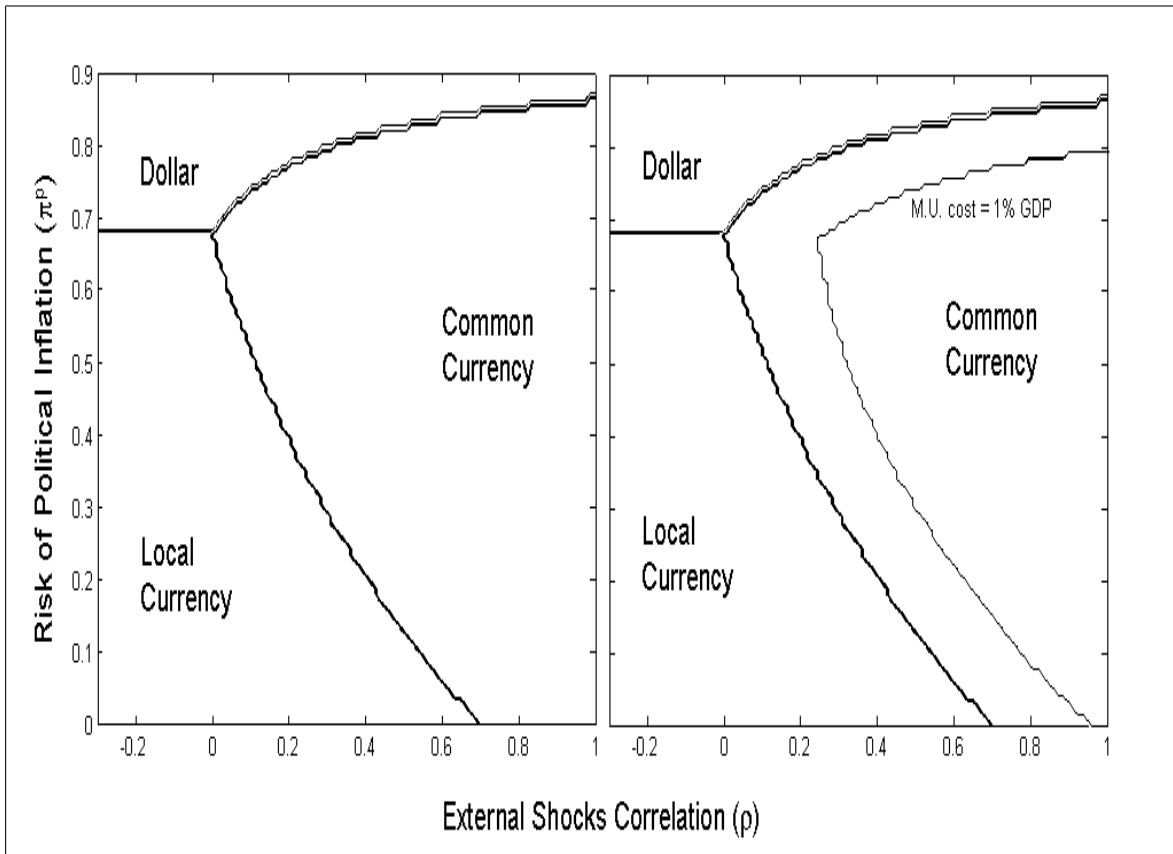


Figure 3: Optimal Monetary Regime (veto allowed, $n=2$)

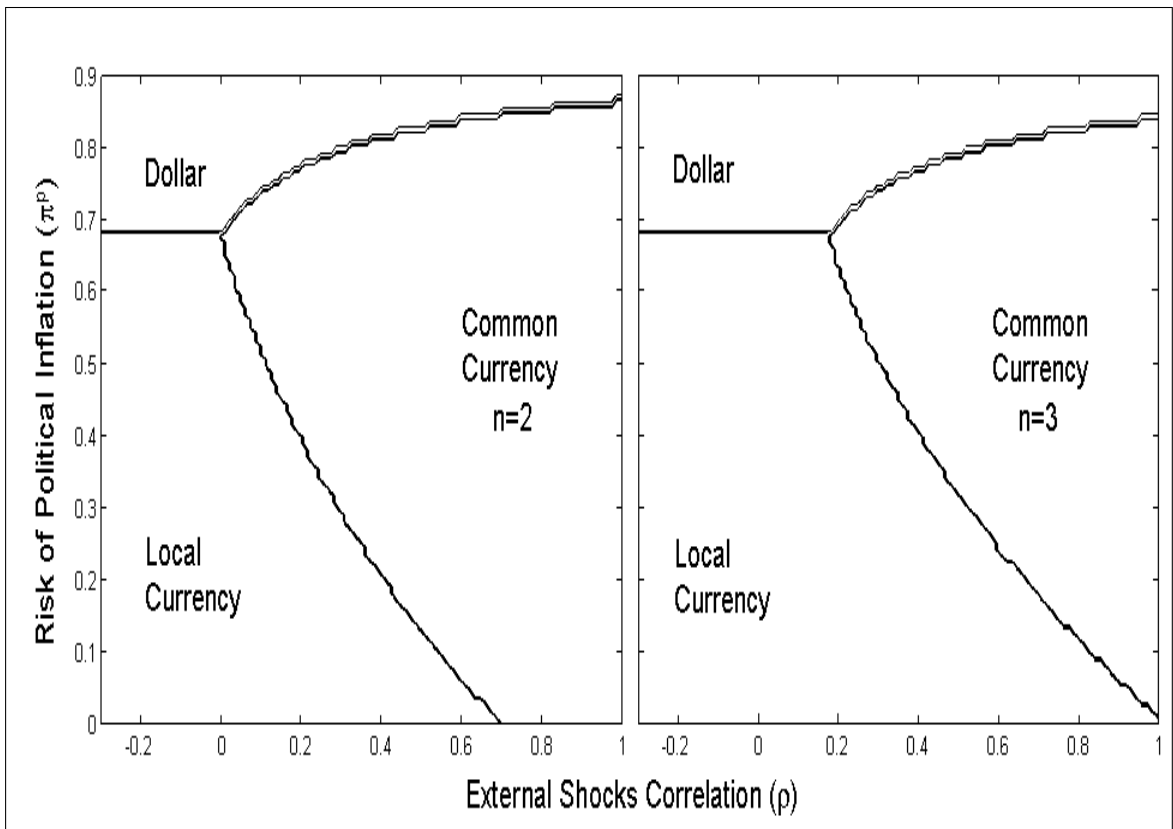


Figure 4: Optimal Monetary Regime (veto allowed, $n=2$ and $n=3$)

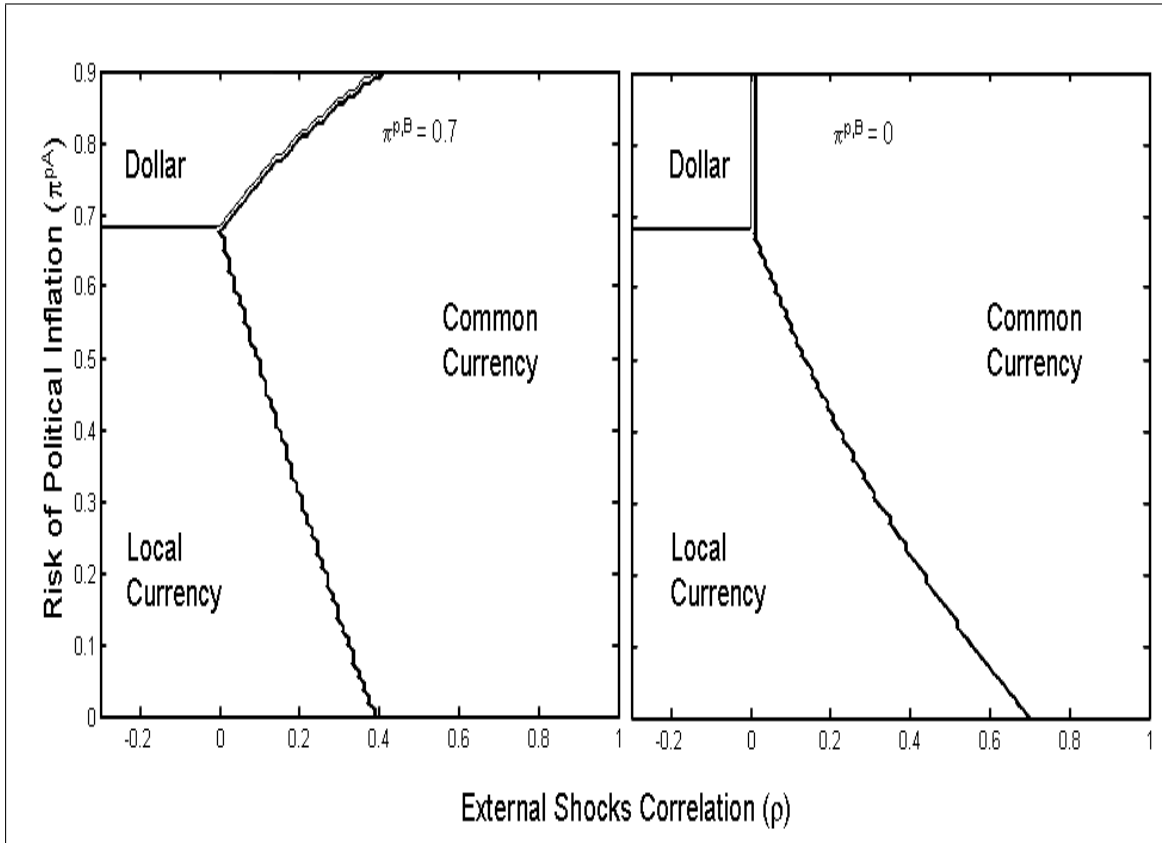


Figure 5: Optimal Monetary Regime ($n=2$, veto allowed, Different π^{pB})

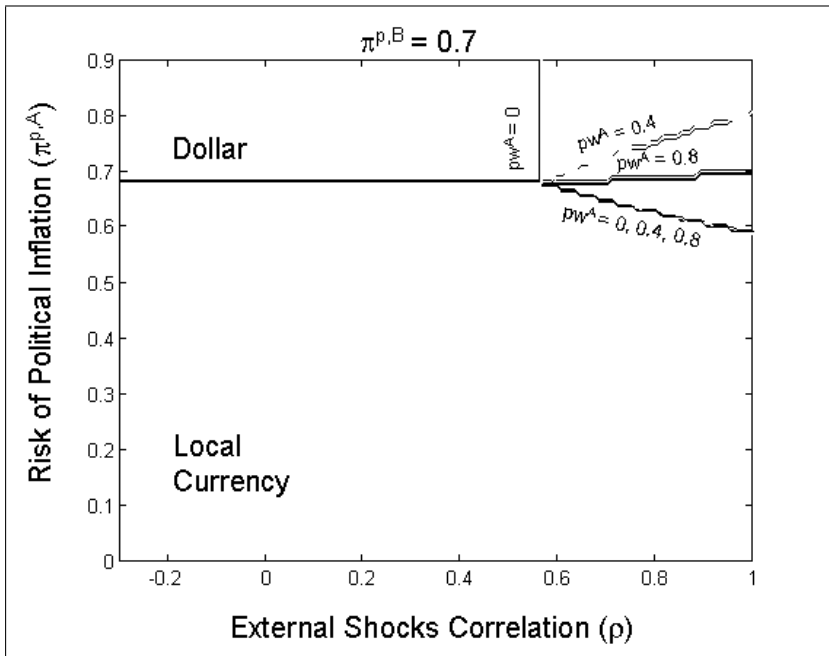


Figure 6: Monetary Union of Members With Different p_w

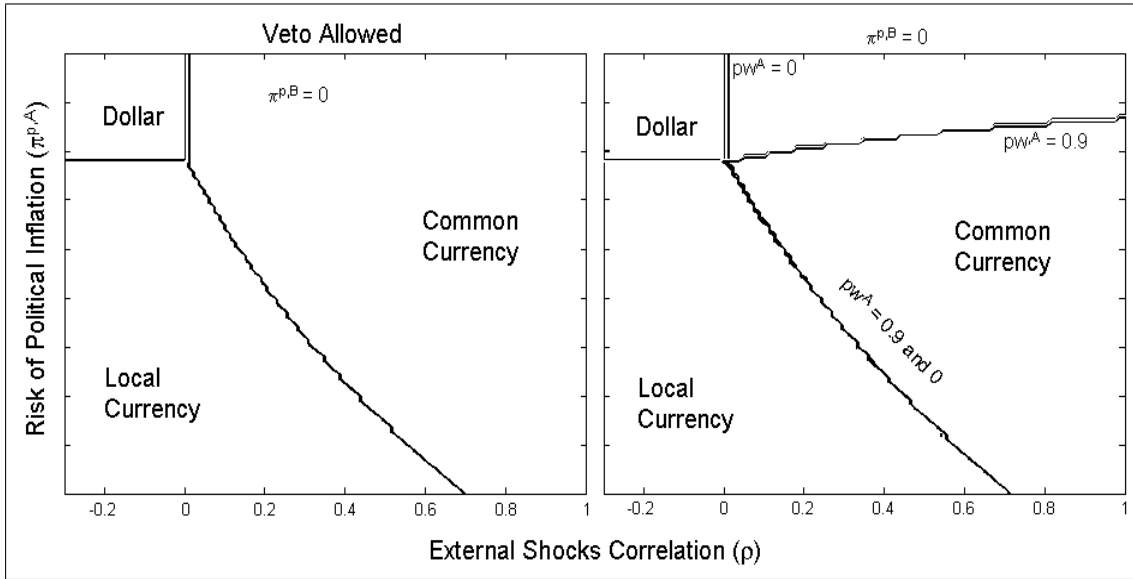


Figure 7: Right of Veto versus Political Influence

2 Speculative Attacks, Openness and Crises

In this paper we propose a dynamic stochastic general equilibrium model to evaluate financial adjustments that some emerging market economies went through to overcome external crises during the latest decades, such as default and local currency devaluation. We assume that real devaluation can be used to avoid external debt default, to improve trade balance and to reduce the real public debt level denominated in local currency. Such effects increase the government ability to deal with external crisis, but also have costs in terms of welfare, related to expected inflation, reductions in private investments and higher interest to be paid over the public debt. We conclude that openness improves expected welfare as it allows for a better devaluation-response technology against crises. We also present results for 32 middle-income countries, verifying that the proposed model can indicate, in a stylized way, the preferences for default-devaluation options and the magnitude of the currency depreciation required to overcome 48 external crises occurred as from 1971. Finally, as we construct our model based on the Cole-Kehoe self-fulfilling debt crisis model ([7]), adding local debt and trade, it is important to say that their policy alternatives to leave the crisis zone remains in our extended model, namely, to reduce the external debt level and to lengthen its maturity.

2.1 Introduction

Both currency and external-debt crises occurred in the latest decades gave new strength to the academic debate on the best exchange rate regime and alternative policies for emerging market economies, which were strongly affected by sudden reductions in the international capital inflows. Studies have associated unexpected financial shocks with currency crises, current account adjustments and default. Cole and Kehoe ([7];[8]; and [9]) developed a model where an indebted country was vulnerable to the willingness of the external creditors to keep its debt rolling, and applied it to the Mexican crisis. Calvo, Isquierdo and Talvi ([5]), based on the Argentina crisis, linked the *sudden stop* events with current account adjustments, currency devaluation and default. They also suggest that the damage associated with the *sudden stop* and the effectiveness of those responses vary between countries, depending on the previous degree of dollarization¹⁷ and openness.

Dollarization mechanisms were large used by emerging economies, specially as from the end of the 80's. The supply of international capital available and the low credibility of the local currencies favored the adoption of price stabilization policies based on the fixed exchange rate, as the currency board in Argentina and Real-Dollar pegged in Brazil. Some economists have pointed out that emerging economies should sustain a really fixed exchange rate regime because their difficulty in conducting appropriate monetary policy with credible local currency ([4],[11], [14]). Other studies have argued that fixed exchange rates do not improve fundamentals and so, it may be just a delay mechanism for intense crises ([6], [15], [19]). Finally, there are studies suggesting that different monetary policies can be adequate to different realities and that each country must find its own solution according to its peculiarities ([1], [2], [13], [16]).

Although there was some disagreement about the best exchange rate regime for emerging countries in the past, now there is some agreement that the more indebted, dollarized and closed the economy is the greater is its vulnerability to sudden reversals in the capital inflows. Reversals induce balance of payments crisis that may be solved by adjustments in the local currency price. This way, a higher degree of openness helps the economy to react through local currency devaluation which improves net exports and consequently smooths the sudden stop. But depending on the intensity of the crisis and on the effectiveness of the currency devaluation, even default can be desired to overcome the external constraint.

This paper aims at evaluating these issues considering 48 crises, occurred in 32 middle-income

¹⁷To be "dollarized" means to be exposed to the exchange rate movements, which can increase obligations in foreign currency assumed previously.

countries, as from 1971. We follow Reinhart, Rogoff and Savastano ([18], tables 3 and 13) to select 31 middle income countries which with Singapore complete our sample¹⁸. Such economies had a significant portion of their external debt denominated in foreign currency when they found themselves in trouble because their inability to obtain new credit in the international market. We can also say that big currency devaluation was used during most of their crises, if we consider “big” a two-digits monthly exchange rate devaluation. All the 48 devaluations considered take place next to default event or after reasonable flat exchange rate period. Finally, the high risk premium observed in the transactions involving their foreign currency indexed bonds suggests that markets are aware that default could be occasionally used.

To study such currency crises including issues as the risk of default, the risk of devaluation and the degree of openness, we extend the self-fulfilling debt crisis model of Cole and Kehoe ([7]). Our model takes into account the effect of a real devaluation on the trade balance of goods. We suppose that, during times of intensive borrowing in the international financial markets, the indebted country imports heavily and, at times of scarce international credit, it makes adjustments to its trade balance to pay for previous indebtedness. Then, we consider that a real devaluation can avoid default on external debt, improve trade balance and reduce the real public debt denominated in local currency through inflation. Such effects increase the government ability to deal with external crises. The magnitude of the optimal devaluation and its effectiveness depend on the intensity of the shock, on the degree of openness and on the pass-through coefficient. The openness affects the response of the trade sector to a change in the relative prices, and the pass-through affects the inflation rate. We also consider that the possibility of a real currency depreciation generates costs in terms of welfare, related to expected inflation, reduction in private investments and higher interest to be paid over the public debt denominated in local currency.

In order to estimate the effect of the openness and debt levels over the past currency crises, we present in Figure 1 the currency depreciation¹⁹ observed versus the degree of openness²⁰ and versus the external debt related taxes for the selected countries. We also present the best linear proxies for both relations. Looking to this plot, we can guess that in most open countries the devaluation required tends to be lesser, even without considering many other important variables in such an analysis.

The following figures present results of the model simulation which are detailed in section 3. They are divided into two blocks. In the first one we use Brazil as a benchmark economy to evaluate some qualitative results from the model, like the optimal fiscal policy function; the crisis zone; how the degree of openness can affect this zone and the devaluation-response; how much degree of openness is required to compensate the negative welfare effect of more vulnerability (more risk premium and/or more external debt); what should be the shock required for the default to be better than the devaluation; and how the degree of openness affects this requirement. In the second, we show aggregate results for all countries, verifying that our model can indicate, in a stylized way, the preferences for default-devaluation options and the magnitude of the currency depreciation required to overcome external crisis.

On a more methodological ground, the possibility that default can be welfare enhancing is in accordance with the current bankruptcy literature, which says that it is optimal to have some bankruptcy in equilibrium, contrary to conventional wisdom (see Geanakoplos, Dubey and Shubik [12], for penalties on the utility function, and Araujo, Páscoa and Torres-Martínez [3], for infinite horizon economies). Although, the risk of default should be kept under control. Accordingly, the introduction of local currency and tradable goods can give rise to the possibility of a better bankruptcy technology through devaluation of the local currency than just the repudiation of the external debt, which can be

¹⁸See Table 3. Iran, Lebanon, Panama, Peru and Poland are not considered in our sample because some of their information was not available.

¹⁹We present details of these calculations and plot in the numerical exercise section and in the Table 3. Note that only devaluations bigger than 30% are selected.

²⁰The degree of openness is defined as $(\text{Imports} + \text{Exports}) / (\text{GDP})$.

quite costly.

In the next section we describe the model and an equilibrium. Section 3 presents the numerical exercises and section 4 concludes them.

2.2 The Model with Tradable

The self-fulfilling debt crisis model of Cole and Kehoe ([7]) including debt denominated in local currency is the basis to develop the model with trade. There is only one good in the economy produced with capital, k , inelastic labor supply and price normalized to one; three participants — national consumers, international bankers and the government; public external debt denominated in dollars or indexed to this currency, B^* , and public internal debt denominated in local currency, B . The external public debt is only acquired by international bankers, and there is a positive probability of no rollover whenever its level is in the crisis zone. We consider that any suspension in its payment is permanent and total, as in the original model. On the other hand, public debt B is only taken up by national consumers, which are always prone to rolling it over, charging the price associated with the positive probability of partial repayment due to currency devaluation.

2.2.1 Uncertainty

To characterize uncertainty, we suppose that sunspot variables give a representation of real shocks and not just psychological ones. The model contains two sunspots: one representing the local investors' confidence, ζ , and another portraying external investors' confidence in the government, ζ^* , which is supposed to be distributed with uniform $[0;1]$. Even though these shocks are not explicitly modeled, the fear of a sudden fall in a commodity price that represents a big share of exports, of changing in the government preferences about public expenditure, or a political turmoil which contributes to the reversal of international capital flows are examples of shocks that the sunspot variables ζ^* and ζ aim at representing. To generate extra revenues in response to a reversal of capital inflows the government can default on the dollar debt or else, it might devalue the local currency. Therefore, the realization ζ^* indicates the confidence that international investors have that the government will not default on the dollar debt. Likewise, the sunspot ζ describes the national investors' confidence that the government will not devalue the local currency.

We consider that the sunspot ζ realization is conditional on ζ^* realization. The probability that the international bankers' confidence is below the critical value π^* is defined as π^* , *i.e.* $P(\zeta^* \leq \pi^*) = \pi^*$. If $\zeta^* \leq \pi^*$, ζ is supposed to be distributed with uniform $[0;1]$, but if $\zeta^* > \pi^*$, ζ is supposed to be one. The probability that the consumers' confidence is below the critical value π given that international bankers are not willing to renew their loans is π , *i.e.* $P[\zeta \leq \pi \mid \zeta^* \leq \pi^*] = \pi$. Then, in the model with debt in local currency, the probability of a self-fulfilling external debt crisis occurring is $\pi^*(1 - \pi)$, which is equal to $[P(\zeta^* \leq \pi^*)] \cdot [P(\zeta > \pi \mid \zeta^* \leq \pi^*)]$. In this case, there is a suspension of foreign credits and the price that the international bankers are willing to pay for the new dollar debt, q^* , is zero. The fear of default is self-fulfilling. With probability $\pi^*\pi$ there is an external crisis, $q^* = 0$, but the government devalues its currency in order to avoid the default. In this case both national and foreign creditors have very low confidence that the government will honor its debt obligations. Figure 2 sums up the three possible states in the crisis zone.

We rule out from the model the possibility that both default and devaluation can be used together. Instead, we consider that during a crisis each one of them can be used with some positive probability, given by sunspot, ζ . These probabilities can be inferred from the spreads of the interest rates on local currency public debt and on dollar currency public debt over the free risk rate. The results presented in the numerical solution (Figure 7) show that for low levels of the external debt, devaluation is the best response in the crisis zone and, for high levels, default is preferable. We also assume that the commitment to no-devaluation (no-inflation) is enforceable when there is no external crisis ($q^* > 0$).

This way, devaluation can be used only to avoid default, when the economy is hit by a shock.

2.2.2 Crises responses and trade openness

The decision to default on the dollar debt is characterized by the government's decision variable, z^* , being equal to zero from default decision on, or being otherwise equal to one. We assume that default causes a permanent fall in national productivity, a , from 1 to α , with $\alpha \in (0, 1)$. Meanwhile, the decision whether or not to devalue the local currency is described by the government decision variable, z , with $z \in (0, 1]$. On one hand, when international creditors do not renew their loans and the government chooses to default, the local-currency bond pays one good, $z = 1$, and the dollar bond pays nothing, $z^* = 0$. The productivity of the economy falls to α . On the other hand, if the government decides for devaluation, then local-currency bond delivers ϕ goods, $z = \phi$, there is no default on the dollar debt, $z^* = 1$, and the productivity turns to be α^ϕ . The cost of devaluation is described by a permanent fall in productivity, a , from 1 to α^ϕ , with $\alpha^\phi \in (0, 1]$ for all ϕ . Note that ϕ corresponds to the best inflation response against the crisis and conditional on $z^* = 1$.²¹ Therefore, devaluation of the local-currency also brings a cost in terms of lower productivity and a benefit of extra revenue that helps to avoid an external default. Finally, provided that there has not been either a default or an inflation tax, previously or at present, then a is equal to one and the government decision variables are $z^* = 1$ and $z = 1$. Figure 2 shows the three possible government decisions and implied productivity, depending on the realization of the sunspots and considering external debt in the crisis zone. The crisis zone is defined as the interval of the external debt for which the government prefers to default if ($q^* = 0$), and not to default if ($q^* > 0$). Both equilibria are possible and the selected one is given by the sunspot variables.

• Real devaluation

The international market is the place to settle dollar-denominated debt, while the domestic market is the one to settle local-currency obligations. The indebted country produces goods locally and pays the maturing external debt according to the price of the good exchanged abroad, which we call tradable. There is only one good but with different prices depending on the place it is traded. This trade could be thought of as occurring through an exchange rate market. The government budget constraint in each period (t), in units of the domestic goods, is given by:

$$g_t \leq \theta \cdot [a_t f(K_t) - \delta K_t] + TB_t - R_{t-1} z_t^* B_t^* + R_t q_t^* B_{t+1}^* - B_t z_t + B_{t+1} q_t \quad (1)$$

with g being the public expenditure, θ being the tax rate, $f(\cdot)$ ²² being the production function, K is the capital stock of the economy, R is the amount of domestic goods per unit of tradable, δ is the depreciation factor, TB_t is the trade balance and q, q^* are the prices of the local currency-denominated bond in unit of domestic goods and the dollar-denominated bond in units of tradable, respectively.

We suppose that every country's international transaction occurs through the government budget constraint (1). This way, the imbalances in the current account, $(-TB_t)$, plus the imbalances in the nonreserve capital account, $(R_{t-1} z_t^* B_t^* - R_t q_t^* B_{t+1}^*)$, are compensated by official reserve transactions, which cause a reduction in the public expenditure (g). We also assume that the nominal exchange rate is fixed or pegged to another currency, but might suffer a significant devaluation after the realization of an external shock. National governments may choose to devalue the local-currency in order to make local goods cheaper, impacting the trade balance, TB_t , and the return of its debts. As long as

²¹We consider that this best-response is also permanent, i.e. if $z_t = \phi < 1$ then $z_{t+i} = \phi \forall i \geq 0$. Then, there is a permanent and constant inflation rate equal to $\left(\frac{1-\phi}{\phi}\right)$ after devaluation. Real effects over local public debt return and real exchange rate is present in the first period after devaluation. From the second period on, inflation is predictable and affects only nominal variables.

²²Continuous, concave, differentiable and strictly increasing; $f(0) = 0$; $f'(\infty) = 0$; $f'(0) = \infty$

the pass-through coefficient from nominal exchange rate to prices, τ , is less than one, the devaluation is followed by a rise in the real exchange rate, R , and a rise in the domestic inflation, which implies that z is less than one, *i.e.* $z = \phi(\tau)$. In this case, the government pays ϕB to local investors, reducing the real return on the local-currency debt. Furthermore, the real devaluation increases the volume of exports and decreases the demand for both imports and new dollar debt. We consider that it does not increase the price to be paid for the old external debt as the government can pay it before changing R .

Then, the advantages of the devaluation-response embrace avoiding an external default, reducing the local-currency debt to GDP ratio; since ϕB , instead of B , is settled from the moment of the devaluation on; and improving the trade balance. All these gains should be weighed up in terms of welfare. On the other hand, there are costs related to the inflation tax and to the rise in the value of the foreign obligations.

To compute such effects in the trade balance, we consider that its value, $TB(R)$, depends on the real exchange rate. At times of no external crisis, R is equal to one and the trade balance enters as a constant term in the government budget constraint. We are only interested in the revenue that the government obtains from an improvement in the trade balance after a real devaluation, $D(R)$ ²³. This revenue depends on the intensity of the real devaluation, the trade volumes, and the real-exchange-rate elasticities of exports and imports, η and η^* , respectively²⁴, as developed in the Appendix. We set $D(R)$ as

$$D(R) = (R - 1) (\sigma\eta + \eta^*R - 1) Imp(1)$$

where $(R - 1)$ is the rate of devaluation, σ is the export-import ratio and $Imp(1)$ is the initial level of imports when $R = 1$. Then, a devaluation produces a positive change in the trade balance as long as $(\sigma\eta + \eta^*R)$ is greater than one, which means that the trade account is improved when the response of export-import ratio to a change of the real exchange rate is preponderant. In this case, the more price-elastic the trade volumes are, the greater the improvement will be. Note that the devaluation also may worsen the trade account because of the negative wealth effect on the import volumes ordered before the change in the real exchange rate.

2.2.3 Market participants

At any time t , the representative consumer maximizes the expected utility

$$\max_{\{c_t, k_{t+1}, b_{t+1}\}_t} E \sum_{t=0}^{\infty} \beta^t [c_t + v(g_t)]$$

subject to the budget constraint, given by

$$c_t + k_{t+1} - k_t + q_t b_{t+1} \leq [a_t \cdot f(k_t) - \delta k_t] (1 - \theta) + b_t - b_t (1 - z_t)$$

given $k_0 > 0$ and $b_0 > 0$. At time t , the consumer chooses how many goods to save for the next period, k_{t+1} , to consume at present, c_t , and the amount of new local-currency debt to buy, b_{t+1} , which consists of zero-coupon bonds maturing in one period. The utility has two parts: a linear function of private consumption, c_t , and a logarithmic function of government spending, $v(g_t) \equiv \ln(g_t)$. The right-hand side of the budget constraint corresponds to the sum of consumer's income from production, after taxes and capital depreciation, plus the return on the local-currency debt acquired in the previous period. If there is no devaluation, z_t equals to one and this return equals to b_t domestic goods.

Analogously, at any time t , the problem of the representative international banker is

$$\max_{\{x_t, b_{t+1}^*\}_t} E \sum_{t=0}^{\infty} \beta^t x_t$$

²³ $TB(R > 1) - TB(1)$

²⁴ $\eta > 0$ and $\eta^* < 0$.

subject to the budget constraint

$$x_t + R_t q_t^* b_{t+1}^* \leq \bar{x} + R_{t-1} z_t^* b_t^*$$

given $b_0^* > 0$. At time t , the bankers choose how many goods to consume, x_t , and the amount of new government bonds denominated in dollar to buy, b_{t+1}^* . The expenditure on new government debt is $R_t q_t^* b_{t+1}^*$, where q_t^* is the price of the zero-coupon bond that pays one unit of tradable good at the maturity $(t+1)$ if the government does not default. The right-hand side includes the revenue received from the bonds purchased in the previous period, $R_{t-1} z_t^* b_t^*$, and the fixed endowment flow, \bar{x} . The decision variable z^* indicates whether the government defaults ($z^* = 0$) or not ($z^* = 1$). If it defaults, then the bankers receive nothing.

The government is assumed to be benevolent in the sense that it maximizes the welfare of national consumers, with no commitment to honor its obligations. Its budget constraint is given by (1), where the left-hand-side is the government's consumption and the right-hand-side includes the following terms: the income tax, the trade balance and the interest paid both on the dollar debt and on the local-currency debt.

In order to obtain the real exchange rate as a function of the government inflation decision, we define the real exchange rate devaluation as:

$$\frac{\Delta R}{R} = \frac{\Delta E}{E} + \frac{\Delta P^*}{P^*} - \frac{\Delta P}{P}$$

Assuming that the foreign price level P^* is constant, we obtain the local-inflation rate \varkappa :

$$\frac{\Delta P}{P} = \frac{\tau}{1 - \tau} \frac{\Delta R}{R}$$

with the pass-through from nominal exchange rate change to local prices, τ , being equal to $(\frac{\Delta P}{P}) / (\frac{\Delta E}{E})$. The value of z , which corresponds to the units of domestic goods that a local-currency bond actual pays at maturity, is defined as

$$z = \frac{1}{1 + \varkappa}$$

Because we consider only the devaluation possibility, i.e. $R_{t+1} \geq R_t$, we have $z \in (0, 1]$. Accordingly, we arrive at an expression that relates z to the change in the real exchange rate:

$$z = \left(1 + (R - 1) \frac{\tau}{(1 - \tau)} \right)^{-1} \quad (2)$$

where the devaluation rate is given by $(R - 1)$, since $R_0 = 1$.

The government is assumed to behave strategically as it can foresee the optimal decisions of all market participants; including its own, z_t^* , z_t and g_t ; given the initial aggregate state of the economy and its choices of B_{t+1}^* and B_{t+1} . To match aggregate and individual variables it is assumed that, in the initial period, the supply of dollar debt B_0^* is equal to the demand for this debt, b_0^* ; the supply of local currency debt B_0 is equal to its demand, b_0 ; and the aggregate capital stock per worker, K_0 , is equal to the individual capital stock, k_0 . The population of both consumers and bankers is continuous and normalized to unit.

2.2.4 A recursive equilibrium

The definition of a recursive equilibrium follows the same procedure developed by Cole and Kehoe ([7]). The actions of the participants are taken backwards according to the timing in each period.

Timing of actions within a period

- the sunspot variables ζ^* and ζ are realized and the aggregate state of the economy is $s \equiv (K, B^*, B, a_{-1}, \zeta^*, \zeta)$;
- the government, taking the dollar-bond price schedule $q^* = q^*(s, B^{*'})$ as given, chooses the new dollar debt, $B^{*'}$;
- the government, taking the local-currency bond price schedule, $q = q(s, B^{*'})$ as given, chooses the new local-currency debt, B' ;
- international bankers, taking q^* and z^* as given, choose whether to purchase $B^{*'}$;
- local investors, considering q^* , q and z as given, decide whether to acquire B' ;
- the government decides whether or not to default on dollar debt, z^* ;
- the government decides whether or not to devalue the local currency, z , and chooses its current consumption, g ;
- consumers, taking $a(s, z^*, z)$ as given, choose c and k' .

Given this timing of actions, we can work backward in each period to define the value functions and the states variables for each market participant, starting with consumers who move last. For simplicity, from now on, we refer to consumers and government decisions as C and G , respectively²⁵.

Each consumer knows the stock of capital saved, k , the stock of local-currency debt purchased, b , the aggregate state of the economy s , and the new dollar and local-currency debts offered by the government, $B^{*'}$ and B' . Given s and $B^{*'}$, consumers take as given the price that international bankers are willing to pay for the new dollar debt, $q^*(s, B^{*'})$, and the price that turns them indifferent to accepting or not the new local-currency debt, $q(s, B^{*'})$. They are also able to anticipate the government's decisions for the coming period, $G(s')$. Therefore, when consumers choose the amount of new local-currency debt, b' , their state is $s_c \equiv (k, b, s, G(s))$. The value function for the representative consumer is given by:

$$V_c(s_c) = \max_{c, k', b'} \{c + v(g) + EV_c(s'_c)\} \quad (3)$$

$$s.t. \quad : \quad c + k' - k + q b' \leq (1 - \theta) [af(k) - \delta k] + zb; (c, k') > 0.$$

Each international banker decides how much public debt $b^{*'}$ to buy, knowing the amount of dollar debt purchased in the previous period, b^* , the aggregate state, s , and the government's decision $G(s)$. They also take prices as given, $q(s, B^{*'})$ and $q^*(s, B^{*'})$, and the government decisions for the coming period $G(s')$. Accordingly, their state is $s_b \equiv (b^*, s, G(s))$ and their value function is defined by:

$$V_b(s_b) = \max_{x, b^{*'}} x + \beta EV_b(s'_b) \quad (4)$$

$$s.t. \quad : \quad x + Rq^* b^{*' \leq \bar{x} + R_{-1} z^* b^*; x \geq 0$$

The subscript $_{(-1)}$ indicates that when $q^* = 0$ and the government choose to devalue, it can pay the old debt first.

Finally, the government makes decisions twice within a period. When it chooses its new debt levels, it knows the state s . Moreover, it takes price schedules $q^*(s, B^{*'})$ and $q(s, B^{*'})$ as given, as well as its optimal choices induced by the new debts, $G(s'|B', B^{*'})$. Likewise, the government also recognizes that it can affect the optimal choices of consumers, $C(s_c)$, the price schedules $q^*(s, B^{*'})$ and $q(s, B^{*'})$, and the productivity $a(s, z^*, z)$ of the economy.

²⁵ $C = (c, k', b')$, $G = (z, z^*, g, B', B^{*'})$

Then, in the beginning of each period, the government chooses B^{*l} and B^l , and its value function is defined by

$$V_g(s) = \max_{B^{*l}, B^l} c(s_c) + v(g) + \beta EV_g(s') \quad (5)$$

$$s.t. \quad : \quad g = g(s_g); \quad z^* = z^*(s_g); \quad z = z(s_g)$$

where the last three restrictions indicate that the government chooses the best actions given its previous choices about the debt level. s_g is defined as the state of the government, (s, B^{*l}, B^l) . Therefore, after national and international investors decide about buying new debt, the government decides whether or not to default and whether or not to devalue the local currency. By comparing welfare levels according to repayments decisions, and taking all debt levels as given, the policy functions $z^*(s_g)$, $z(s_g)$ and $g(s_g)$, are solutions for:

$$\max_{z^*(s), z(s), g} c(s_c) + v(g) + \beta EV_g(s') \quad (6)$$

subject to,

$$g + z^* R_{(-1)} B^* + z B \leq \theta [a(s, z^*, z) f(K) - \delta K] + TB(R) + q^* R B^{*l} + q B^l$$

$$g \geq 0, z \in (0, 1]$$

$$0 = (R - 1)(1 - z^*)$$

where the last restriction indicates that default and devaluation cannot be implemented at the same time. We defined z and z^* as function of s because when to default is better than not to default ($z^* = 0 \succ^g z^* = 1 | z = 1$), then the actual government response to the crisis, namely devaluation or default, depends on the sunspot- ζ realization. We also consider that when devaluation is used to avoid the default, z is chosen to maximize the welfare conditional to $z^* = 1$.

Definition of an equilibrium

An equilibrium is defined as a list of value functions V_c for the representative consumer, V_b for the representative international banker, and V_g for the government; policy functions C for the consumer, b^{*l} , for the international banker, and G for the government; price functions for the dollar debt, q^* , and for the local-currency debt, q ; and an equation for the aggregate capital motion, K' , as follows:

- (i) given G, q, q^* ; V_c is the value function for the solution to the problem of the consumers (3), and C are their optimal choices;
- (ii) given G, q, q^* ; V_b is the value function for the solution to the problem of the international bankers (4), and b^{*l} is their optimal choice;
- (iii) given C, q, q^* ; V_g is the value function for the solution to the problem of the government (5 and 6) and G are its optimal choices;
- (iv) $B^{*l}(s) \in b^{*l}(s_b)$;
- (vi) $B^l(s) \in b^l(s_c)$;
- (vii) $K'(s, G(s)) = k'(s_c)$

2.2.5 Equilibrium Analysis

The behavior of consumers and bankers depends on their expectations regarding whether or not the government will default on the dollar debt or create inflation tax on the local-currency debt through devaluation. On the other hand, the government actions also depend on these expectations which have real effects through the debt prices and investment levels.

When making their decisions about capital accumulation, consumers compute the expected productivity for the economy according to their beliefs about the possibility of a crisis occurring in the next period, and to the possibility that this crisis results in a default or a currency devaluation. Then, the optimal capital accumulation, k_{t+1} , depends on the consumers' expectations about productivity, $E_t [a_{t+1}]$, as follows:

$$\frac{1}{\beta} = 1 + (1 - \theta) [f'(k_{t+1})E(a_{t+1}) - \delta]$$

Furthermore, consumers act competitively and are risk neutral, so they may purchase new public debt denominated in local-currency if its price equals the expected return to $1/\beta$:

$$\frac{1}{\beta} = \frac{E_t [z_{t+1}]}{q_t}$$

The more closed the economy is the greater is the devaluation (inflation) required during the crisis and the smaller is the expected value for z_{t+1} . So, interpreting $\frac{1}{q_t}$ as being the interest factor over the local currency debt we can say that the interest rate is decreasing in the degree of openness.

Analogously, international bankers act competitively and are risk neutral, so they may purchase new public debt denominated in dollar-currency if its price equals the expected return to $1/\beta$:

$$\frac{1}{\beta} = \frac{E_t [z_{t+1}^*]}{q_t^*}$$

During a crisis ($\zeta^* < \pi^*$) they are convinced about default on the next period and so they set $q_t^* = 0$.

Finally, to complete the equilibrium analysis we must find which are the government actions. Letting $V_g(s|z, z^*, q^*)$ denote the payoff to the government conditional on its decisions, z and z^* , and conditional on the price q^* , and also considering that the public debt level denominated in the local currency cannot change over time ($B_t = B_o \forall t$)²⁶, it is possible to construct an equilibrium with the crisis zone defined by the external debt level as in the Cole Kehoe model ([7]).

Next, we define the *participation condition* which ensures that the government will want to honor the current external debt given it is able to sell new one:

$$V_g(s|1, 1, q^* > 0) > V_g(s|1, 0, q^* > 0) \quad (7)$$

Analogously, we define the *no-lending condition* which ensures that the government will want to default during a crisis:

$$V_g(s|1, 0, q^* = 0) > V_g(s|1, 1, q^* = 0) \quad (8)$$

The crisis zone is defined as the interval for the current external debt level, $(b, B]$, where b is the greatest external debt level so as equation (8) does not hold with $(b, k^n) \in s$, and B is the greatest external debt level so as equation (7) holds for $(B, k^{\pi^*}) \in s$. When the economy is out of the crisis zone, i.e. $B^* \leq b$, there is always external credit available. Since the bankers know that the government would not to default even if the price were zero, they do not refuse to rollover new bonds. In the crisis

²⁶From now on we consider that this assumption holds. Otherwise, there would be no external crisis since the absence of new external credit could be compensated by greater local currency debt level.

zone the bankers are aware of the possibility of default, and may refuse to sell new bonds ($q^* = 0$). In this case, default is desirable, but with probability π the government devalues to avoid the default.

Although we do not compare the payoffs of devaluation *versus* default to characterize the crisis zone (we maintain the possibility of both responses occur), we compute in the numerical exercises, presented in the next section, the level of external debt, \tilde{b} , for which government would be indifferent between default and devaluation. We also verify that $\tilde{b} \in (b, B)$ and that devaluation is the best response in the crisis zone if $B^* < \tilde{b}$. If $B^* > \tilde{b}$, default is the best response in the crisis zone.

Table 1 presents the four possible equilibrium capital accumulation and prices, according to the expectations. In the first line of the table we consider that the external debt is out of the crisis zone. In the second one we consider that the external debt level is in the crisis zone and the two last present the values for the after-crisis economy. Each one of the three last lines corresponds to the three states characterized in Figure 2.

2.3 Numerical Exercises

In this section, we first present numerical exercises for the Brazilian economy to show some qualitative results from the model. Secondly, we present aggregate results for 58 currency crises and attempt to outline some of the factors that make countries adopt different crisis responses, with more or less devaluation, and choosing default or not.

2.3.1 Qualitative results

The parameters used in the simulations have been chosen to portray the Brazilian economy during 1998, period that precedes the Brazilian currency devaluation, starting in January 99. The definition of period length is based on the Brazilian government debt whose average length was varied from 7 to 10 months between 98-99. The government discount factor, β , is approximated by the yearly yield on government bond issued by the US, whose values were about 5 percent. Based on these figures, we interpret a period length as being one year and a yearly yield on risk free bonds, r , as being 0.05, which implies a discount factor β of $0.95 (= (1 + r)^{-1})$. The choice of the functional forms for $v(\cdot)$ and $f(\cdot)$ were the same used by Cole and Kehoe [7], that is, $v(g) = \ln(g)$ and $f(k) = Ak^\lambda$ where capital share λ is established at 0.4 and the scale factor at 10. The parameter α equals 0.95, assuming that default causes a permanent drop in productivity of 0.05. For $z = \phi$, the correspondent inflation rate is $(1 - \phi) / \phi$, which implies the welfare cost of inflation, α^ϕ , estimated according to Simonsen and Cysne²⁷ ([10]). The probability of default, $\pi^* (1 - \pi)$, and the probability of inflation, $\pi^* (\pi)$, are calculated on the basis of the risk premium practiced in the financial market according to the following expression:

$$\frac{1}{\beta} = (1 + r_D^{BR}) (1 - \pi^* (1 - \pi)) = (1 + r_{LC}^{BR}) (1 - (\pi^* \pi) (1 - \phi))$$

where r_D^{BR} and r_{LC}^{BR} are yearly yields on Brazilian public debt denominated, respectively, in dollar and in local currency (discounting the expected inflation of Brazilian currency), and ϕ is given by the unexpected yearly inflation associated with devaluation.

Data for r_D^{BR} are available for the period of analysis, while r_{LC}^{BR} only since January 2002, when its value was about 0.12²⁸. Therefore, considering the values for ϕ , r_D^{BR} , and r_{LC}^{BR} equals to 0.5, 0.14, and 0.12, respectively, we can compute (π^*, π) as being equal to (0.2, 0.61). In Table 2 we present the

²⁷In the estimation of welfare cost of inflation we use Bailey's approximation and the money demand specified as $\bar{k}r^{-\bar{a}}$, where r is the logarithmic annual inflation (see Simonsen and Cysne [10]). We set \bar{k} and \bar{a} equals to 0.07 and 0.6, respectively.

²⁸Yearly yield on LTN minus expected inflation.

values of parameters and variables used in the simulations for the Brazilian economy, whose results are described next.

Figure 3 shows that when the external public debt is in the crisis zone the optimal policy is to move out from it. But it may be difficult to reduce public expenditure and Figure 4 shows that an alternative policy could be lengthening the maturity²⁹. These conclusions remain the same as in the original model. Figures 5, 6, and 7 present the effects of the degree of openness over the economy. As shown in Figure 5, if the economy has its imports and exports enlarged without changing the trade balance, i.e. the gains in the volume of exports (Dexp) equal the gains in the volume of imports (Dimp), then only the cap of the crisis zone becomes greater. But if the economy can improve its trade technology and enlarge exports faster than imports, then the international capital inflow becomes greater and both the floor and the cap increases. In figure 6 it is possible to see that, according to the model, the devaluation required to response to a crisis is increasing in the external debt level and decreasing in the degree of openness. Figure 7 also shows that the “devaluation–better-than-default” region is increasing in the degree of openness. Finally, figures 8, 9, and 10 correspond to welfare analyses. They show how much debt must be paid to compensate the welfare loss related to an increase in the external risk, and how much improvement in the trade is required to compensate the welfare loss related to an increase in both the external risk and the external debt. Note that in Figure 10 both expected welfare and welfare after crisis are considered.

2.3.2 Comparative results

The parameters used in the simulations for the other countries are presented in Table 3. To compare results across countries we change only a few parameters which we consider more relevant to explain differences between economies and their responses to crisis. The variables that are not presented in Table 3 are the same for all countries including Brazil (Table 2).

Figure 11 shows that, according to the assumptions of our model, the “countries on the left side” were more prone to choosing default than the “ones on the right”. Results match 85% of the crises with “reality” in predicting that default is the best response whenever it actually occurs and it is not the best response whenever it does not occur. Red marks show where the model failed.

Figure 12 presents the estimated devaluations for different pass-through values. Note that the results are quite similar to a wide range of this parameter. Devaluation rates change significantly only when considering that pass-through is very close to one.

Figure 13 compares actual devaluations and those predicted by the model. In the first plot the elasticities (η^* , η) of 0.6 were considered, and in the second plot we double this value. Note that for greater elasticity less devaluation is required to overcome the external crisis as expected. Accordingly, most devaluations predicted are overestimated, but not too far from reality. Moreover, we do not consider that our numerical exercise is a good predictor for actual devaluation, since we have made simplifications to compute after-crisis payoffs as considering ($z_{t+i}^* = 0$, $z_{t+i} = \phi$) for all $i > 0$, respectively, in default and devaluation responses. Our aim is to outline the different crisis responses adopted by countries considering factors as the degree of openness, debt levels in both currencies, taxes, and risk-premiums. In this sense, we are more interested in comparing the shape of predicted versus actual devaluations plots.

Figure 14 replicates Figure 13 excluding the devaluations lesser than 30% and separating countries that experienced default from the others.

Finally, it is also important to note that there are many ways to compute the actual devaluations of the currencies across countries. We use the exchange rate series published by IMF-Statistics,

²⁹We follow Cole and Kehoe approach for “lengthening the maturity structure”. Henceforth, lengthening the maturity structure means converting an initial quantity B^* of one-period (one year) bonds into equal quantities B_n^* of bonds of maturity n (1,2, . . . ,N). Then, the government redeems B_n^* bonds every period and sells B_n^* n-period bonds, where $B_n^*(1 - q_n^*) = B^*(1 - q^*)$, and $q_n^* = \beta^n(E_t z_{t+n}^*)$.

considering the bilateral price of the dollar related to the local currency, and the mean value for each month. Sudden and significant devaluations indicate the beginning of the crisis period, whose length is defined as six months for all crises. The new level of the exchange rate is considered as the mean of the exchange rate for this crisis period and the actual devaluation for each country is computed as from the exchange rate level immediately before the crisis period.

2.4 Conclusions

As Cole and Kehoe already point out, this type of model differs from most of the literature on debt and currency crises in using a dynamic stochastic general equilibrium framework with an altruistic government rather than using a deterministic model or a model with a reduced form for governments. We extended their debt crisis model, adding trade and local debt, without losing neither the dynamic stochastic general equilibrium framework nor an altruistic government. Some qualitative results like the policy recommendations to leave the crisis zone through policy function and lengthening the maturity of the external debt are the same. Moreover, we can present the welfare effects of the degree of openness and its influence over the crisis zone. The extended model also allows us to study a more realistic economy, which has its own currency. We can predict, in a stylized way, the relative magnitude of the local currency depreciation required to overcome external crises and the preferences for default-devaluation options.

References

- [1] Araujo, Aloisio and Marcia Leon, 2002, Ataques Especulativos sobre Dívidas e Dolarização, *Revista Brasileira de Economia*, 56:1, 7-46.
- [2] Araujo, Aloisio; Leon, Marcia; and Rafael Santos. Monetary Arrangements for Emerging Economies. 2006.
- [3] Araujo, Aloisio; Páscoa, Mário and Juan P. Torres-Martínez, 2002, Collateral Avoids Ponzi Schemes in Incomplete Markets, *Econometrica*, 70:4, 1613-1638.
- [4] Calvo, Guillermo. The case for Hard Pegs. Unpublished manuscript, University of Maryland. Mimeographed.
- [5] Calvo, Guillermo; Izquierdo, Alexandre; and Talvi, Ernesto. Sudden Stops, the Real Exchange Rate, and Fiscal Sustainability: Argentina's Lessons. NBER Working Paper No. 9828. July 2003.
- [6] Chang, R.; Velasco, A.; 2000 . Exchange rate policies for developing countries. *American association papers and proceedings* 90 (May): 71-75.
- [7] Cole, Harold and Timothy Kehoe, 1996, A self-fulfilling model of Mexico's 1994-1995 debt crises, *Journal of International Economics*, 41, 309-330.
- [8] ———, 1998, Self-Fulfilling Debt crises, *Federal Reserve Bank of Minneapolis, Staff Report*, 211.
- [9] ———, 2000, Self-fulfilling Debt crises, *Review of Economic Studies*, 67:1, 91-116.
- [10] Cysne, Rubens and Mario Simonsen, 1994, Welfare Costs of Inflation: The Case for Interest-Bearing Money and Empirical Estimates for Brazil, *Ensaio Econômicos*, 245.
- [11] Dornbush, R.; 2000. Millenium Resolution: No more funny money, *Financial Times*, January 3.
- [12] Dubey, P.; Geanakoplos, J.; and Shubik, M. (2005): "Default and Punishment in General Equilibrium" *Econometrica*, 73(1).
- [13] Frankel, J.; 1999. No single currency regime is right for all countries or at all times. W.P. n 7338, *National Bureau of Economic Research*, Cambridge, Mass.
- [14] Hausmann, R.; 1999. Should there be five currencies or one hundred and five? *Foreign Policy* 116: 65-79.
- [15] Mishkin, F.; 1998. The dangers of exchange rate pegging in emerging market countries. *International Finance* 1: 81:101.
- [16] Mussa, M.; Masson, P.; Swoboda A.; Jadresic, E.; Mauro, P.; Berg, A.; 2000. Exchange rate regimes in an increasingly Integrated world economy. Washington, D.C.: *International Monetary Fund*.
- [17] Paiva, Claudio. Trade Elasticities and Market Expectations in Brazil. *IMF Working Paper*, WP/03/140, Jul. 2003.
- [18] Reinhart, C.; Rogoff, K; Savastano, M.. Debt Intolerance. NBER Working Paper No 9908. August 2003.
- [19] Sachs, J.; Larrain, F.; 1999. Why dollarization is more Straitjacket than Salvation. *Foreign policy* 116: 80-92.

2.5 Figures and Tables

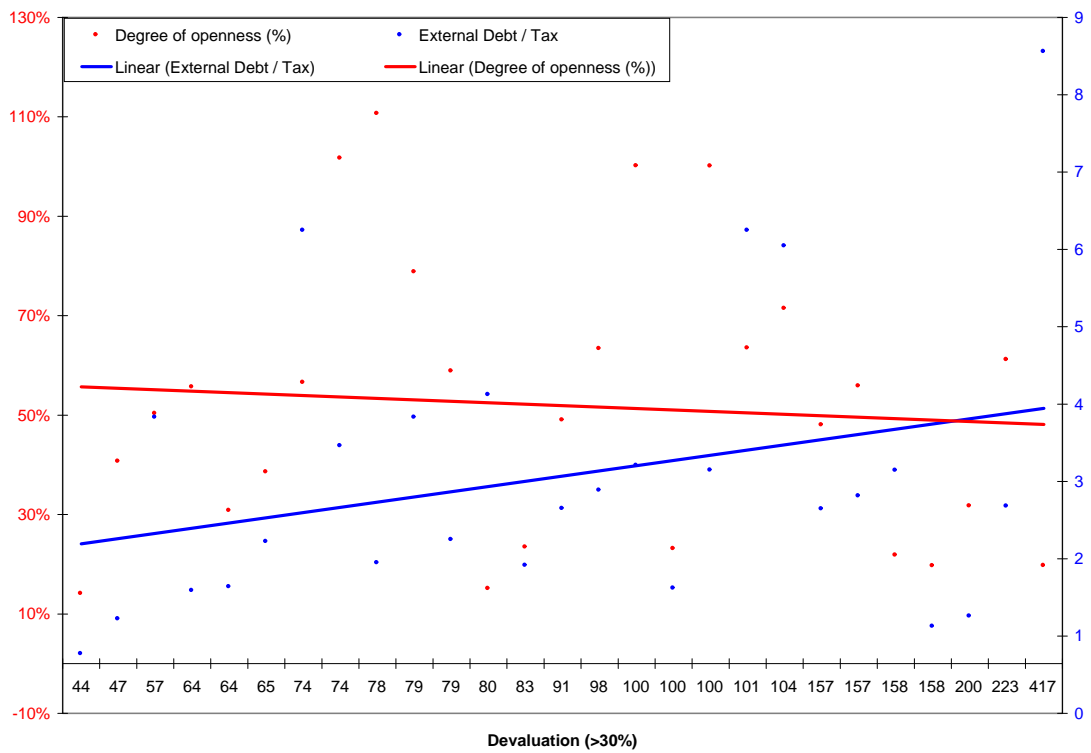


Figure 1: Currency Crisis, Degree of Openness and External Debt

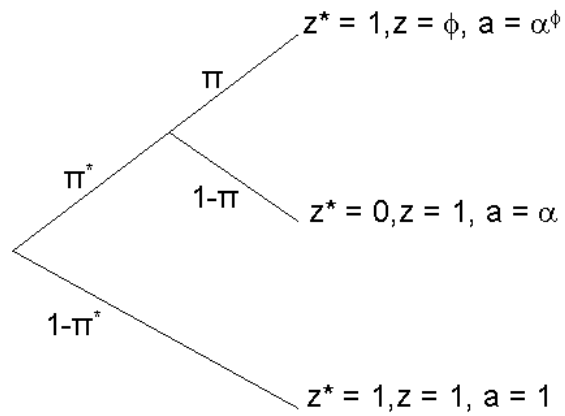


Figure 2: Tree of Events

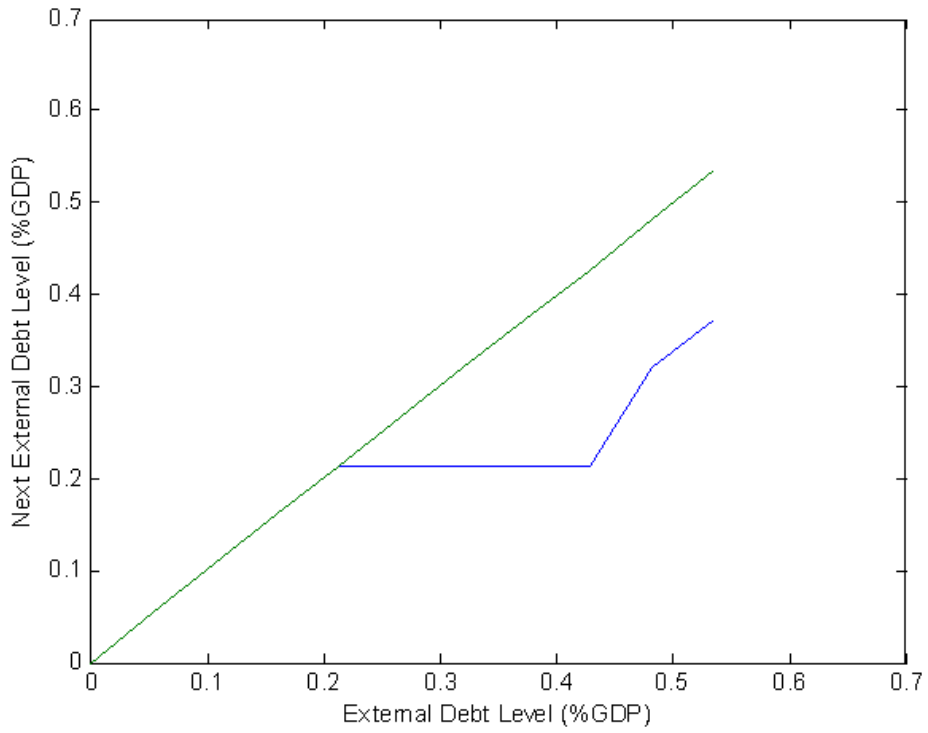


Figure 3: Policy Function - Brazil

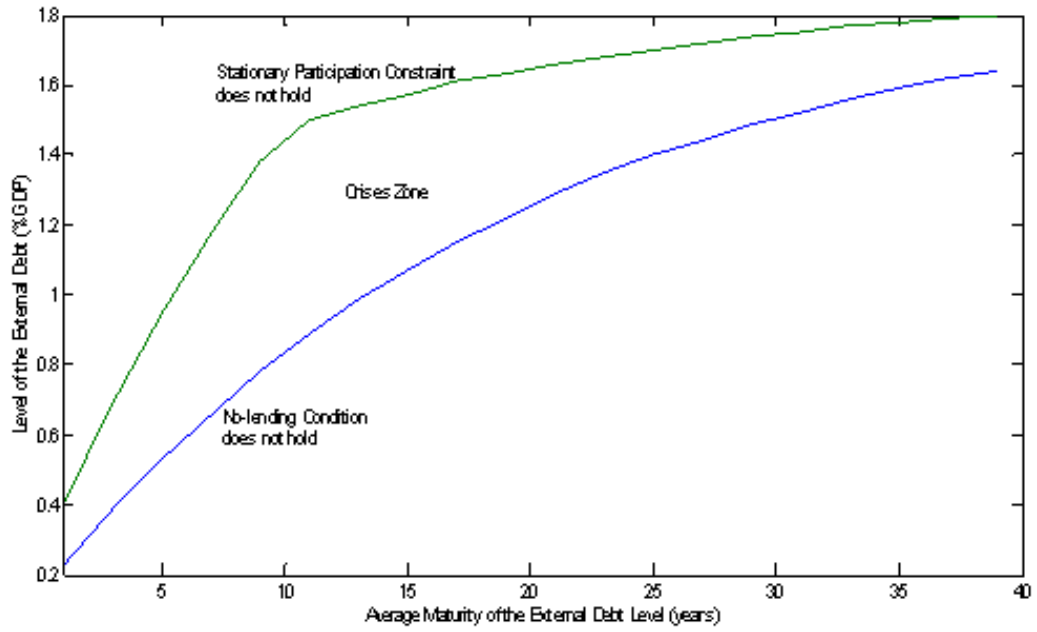


Figure 4: Crisis Zone - Brazil

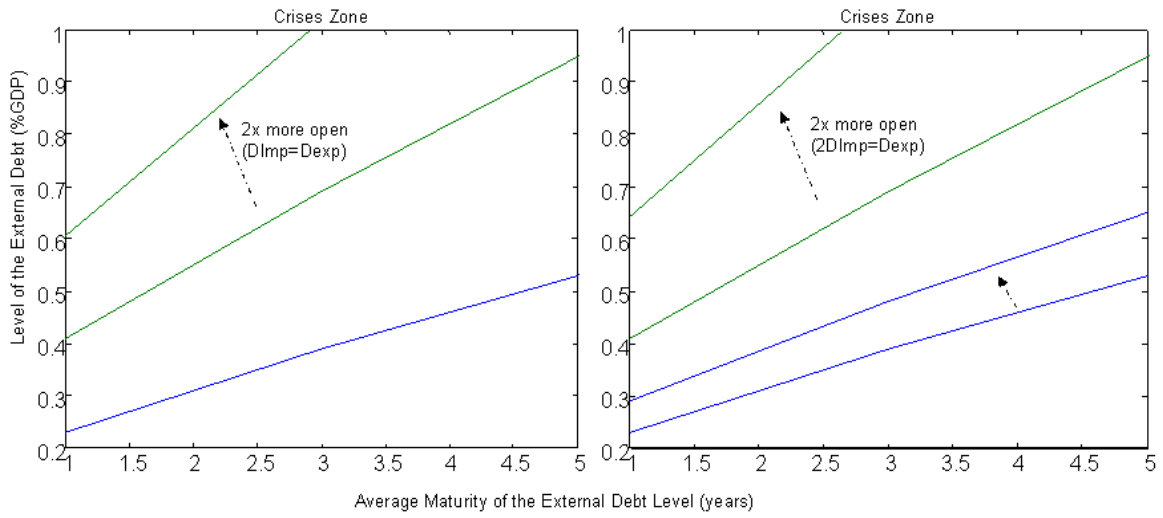


Figure 5: Trade Openness and Crises Zone - Brazil

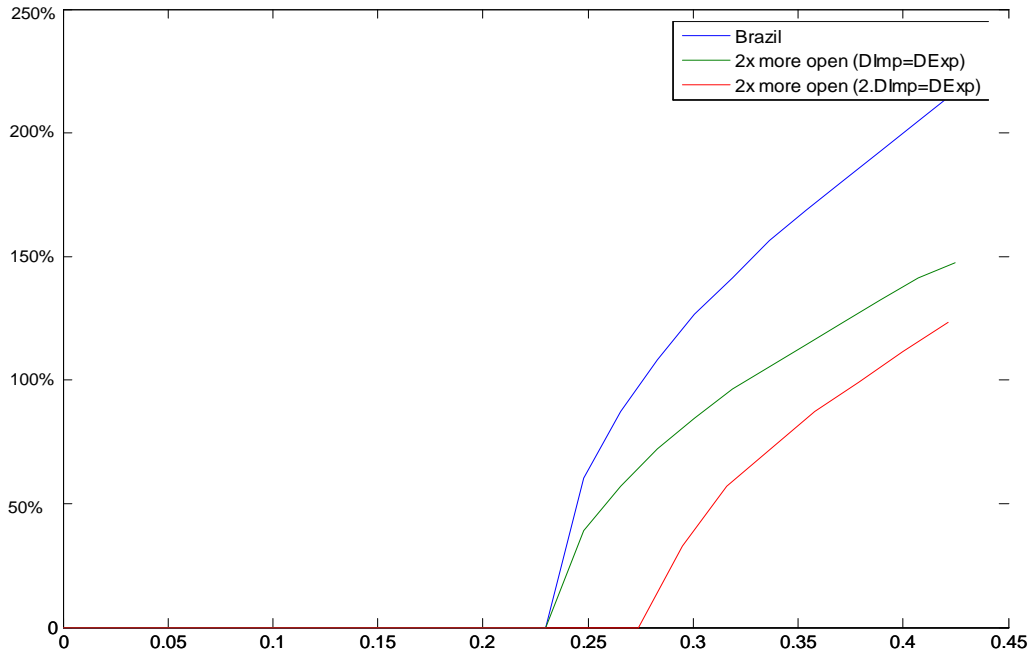


Figure 6: Trade Openness and Devaluation-Response - Brazil

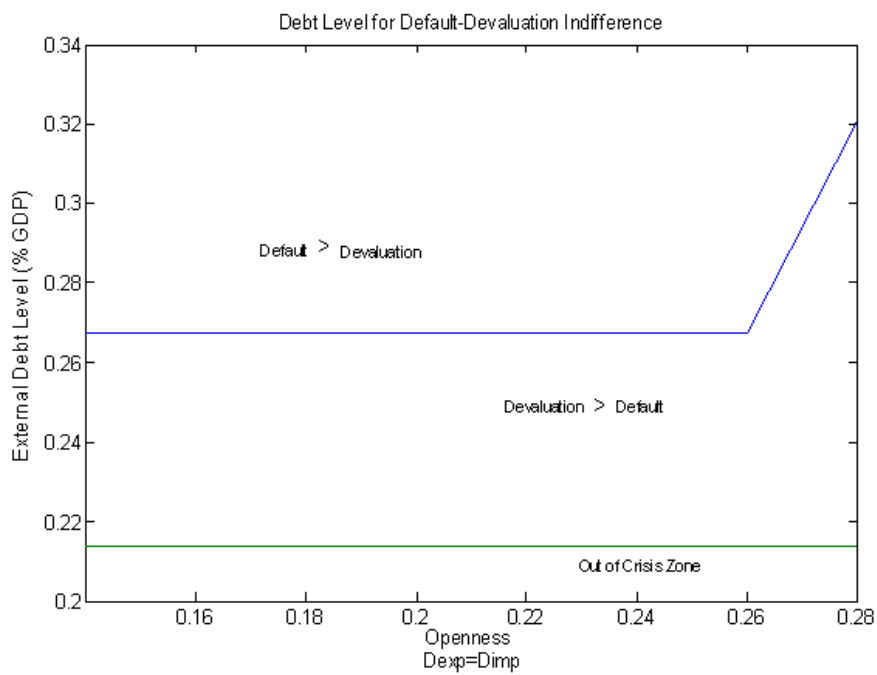
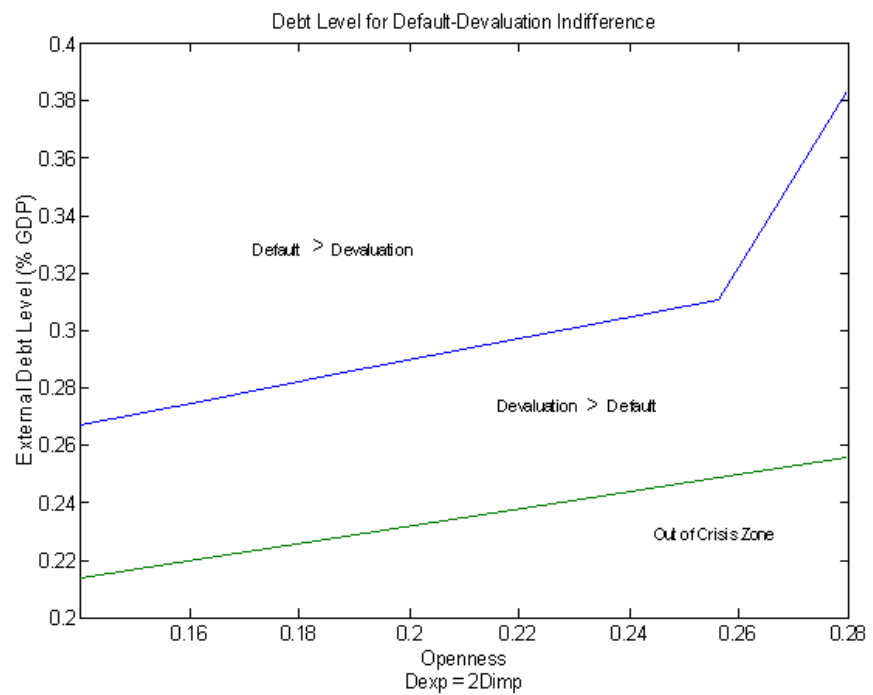


Figure 7: Trade Openness and Default-Devaluation Indifference - Brazil

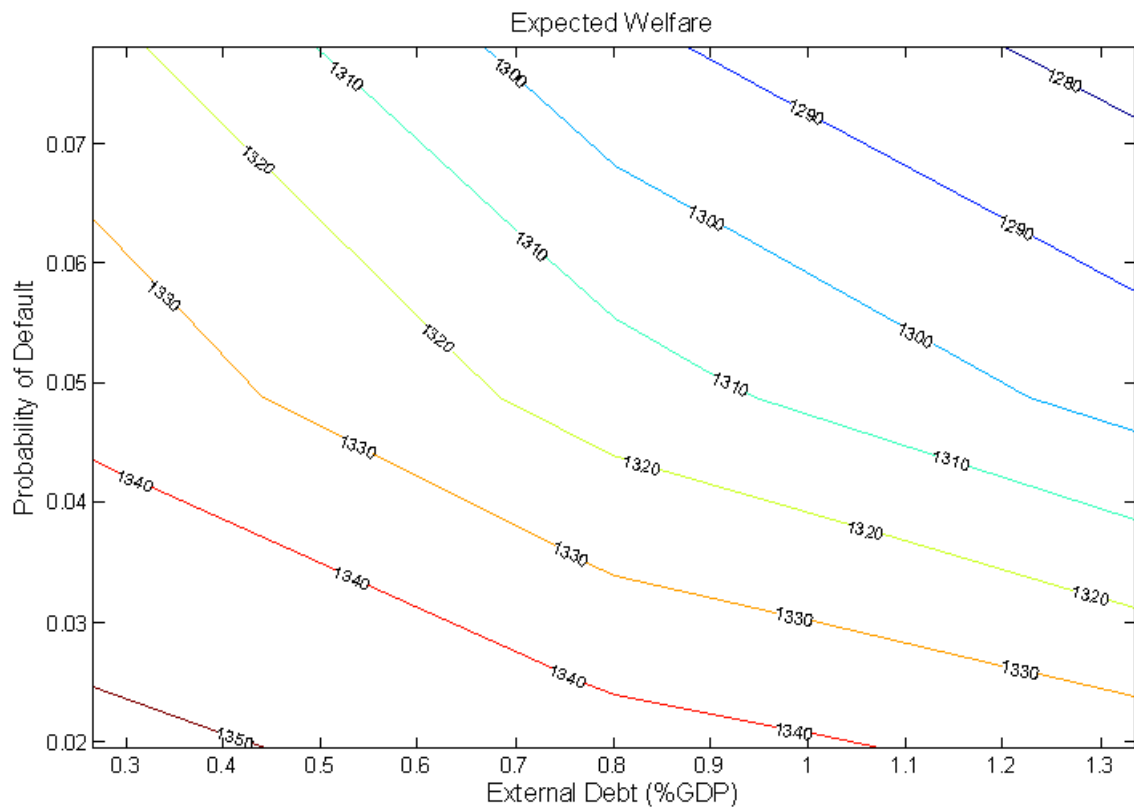


Figure 8: Welfare Sensibility to Shock Intensity- Brazil

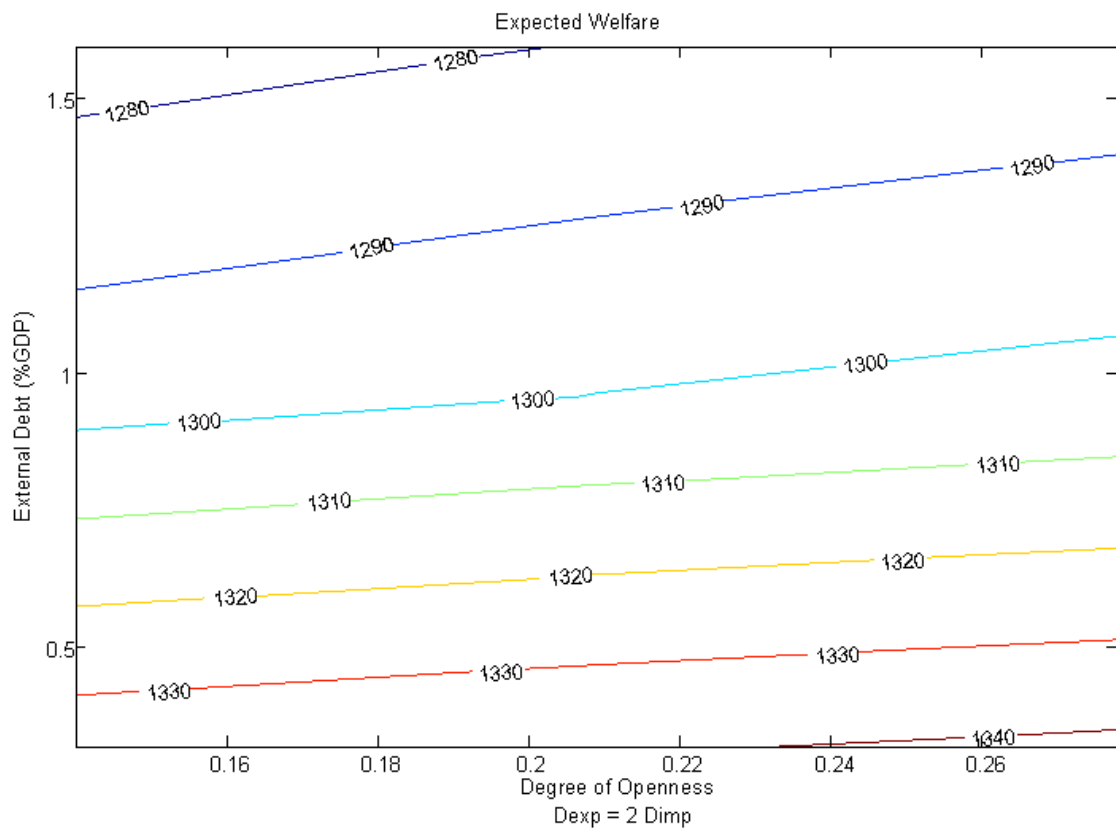


Figure 9: Openness and External Debt Trade-off - Brazil

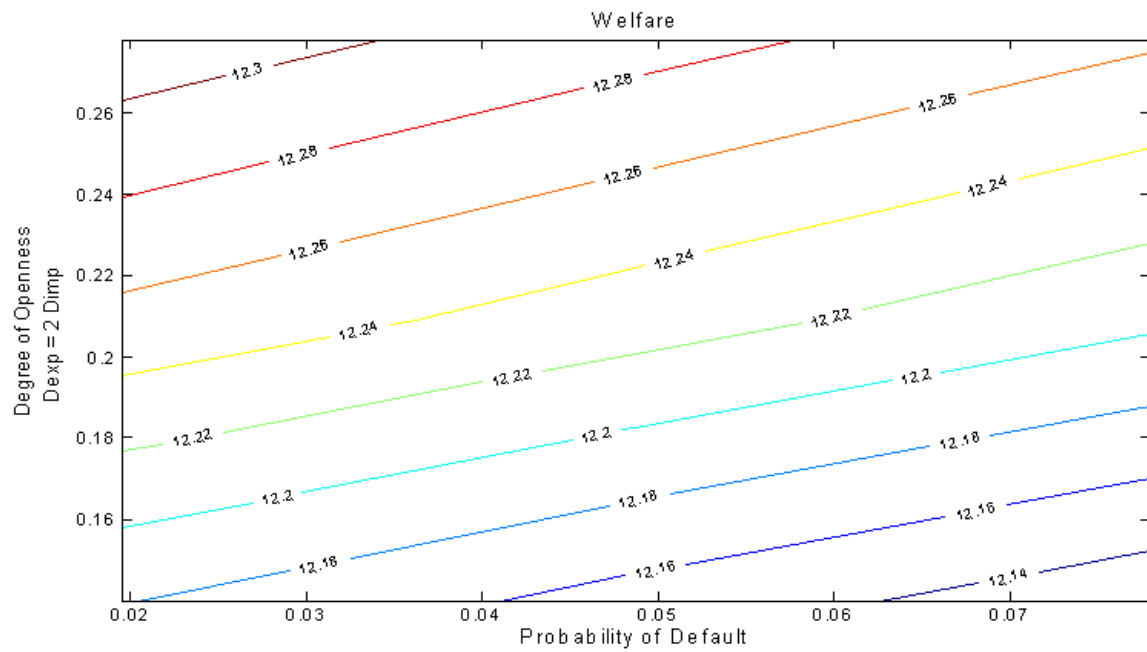
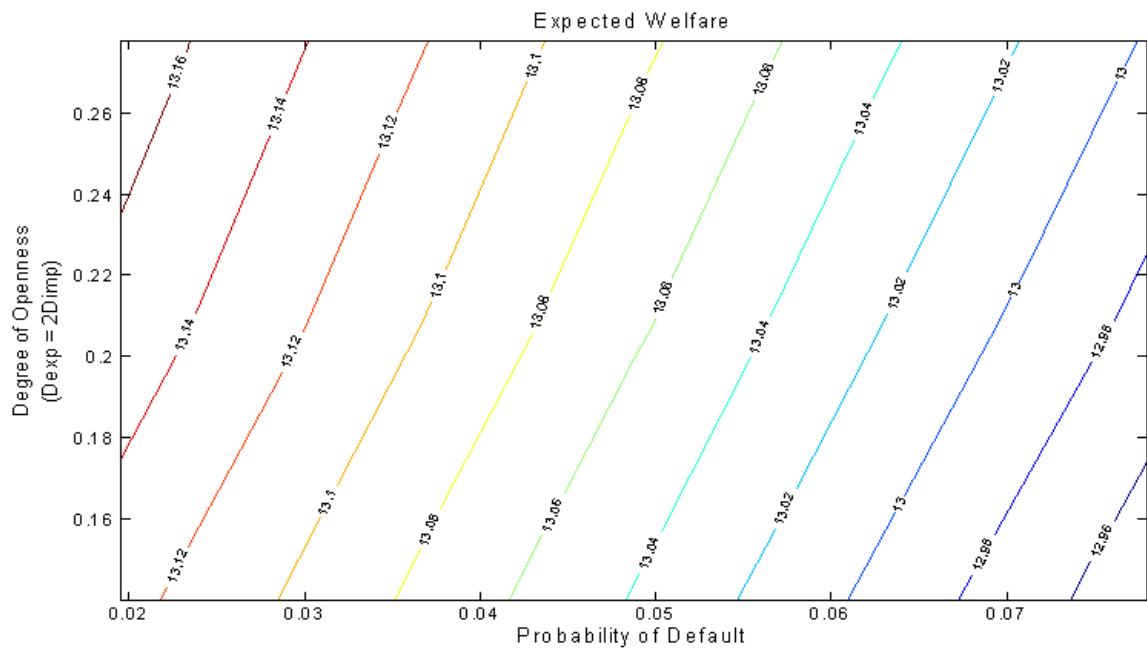


Figure 10: Openness and Risk Premium Trade-off - Brazil

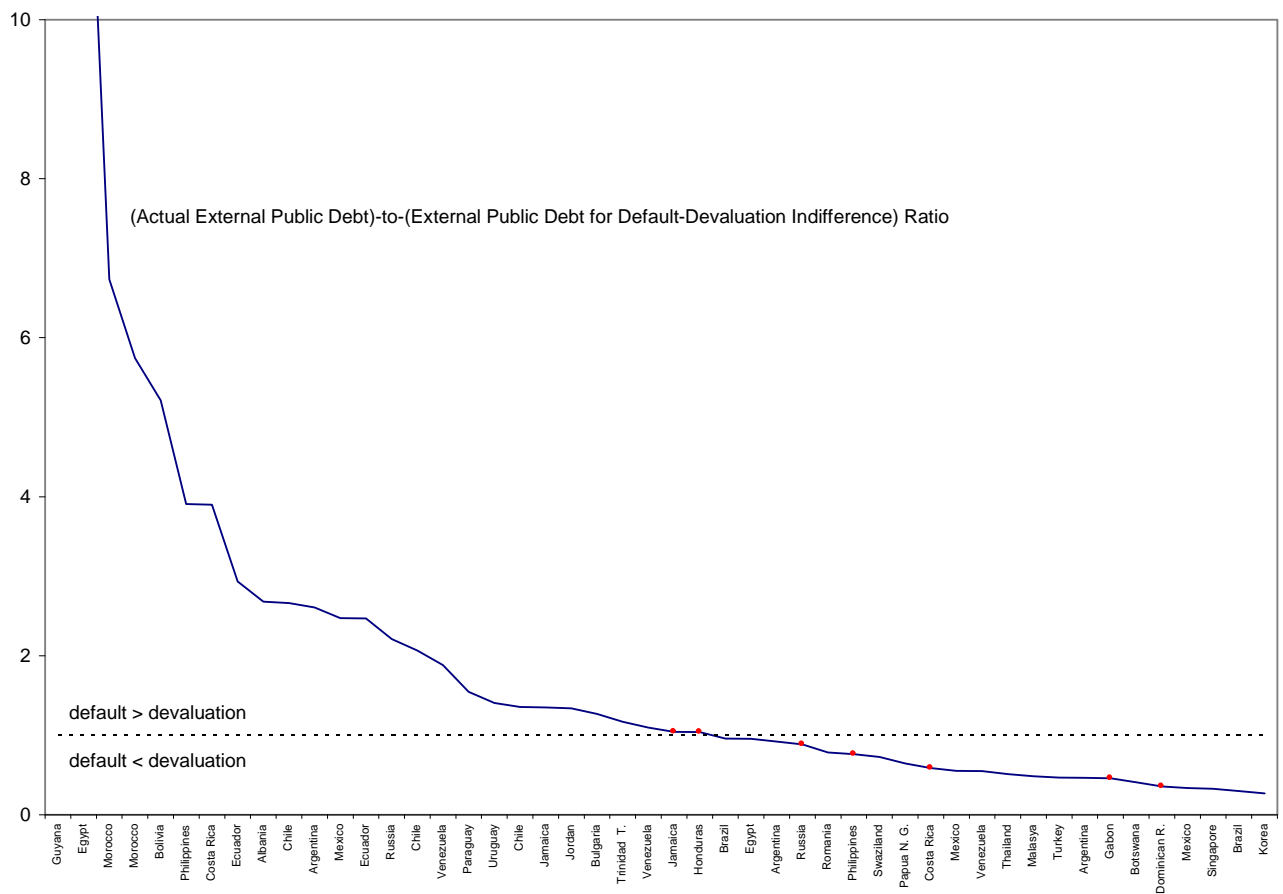


Figure 11: Incentive for Default versus Devaluation in Selected Countries

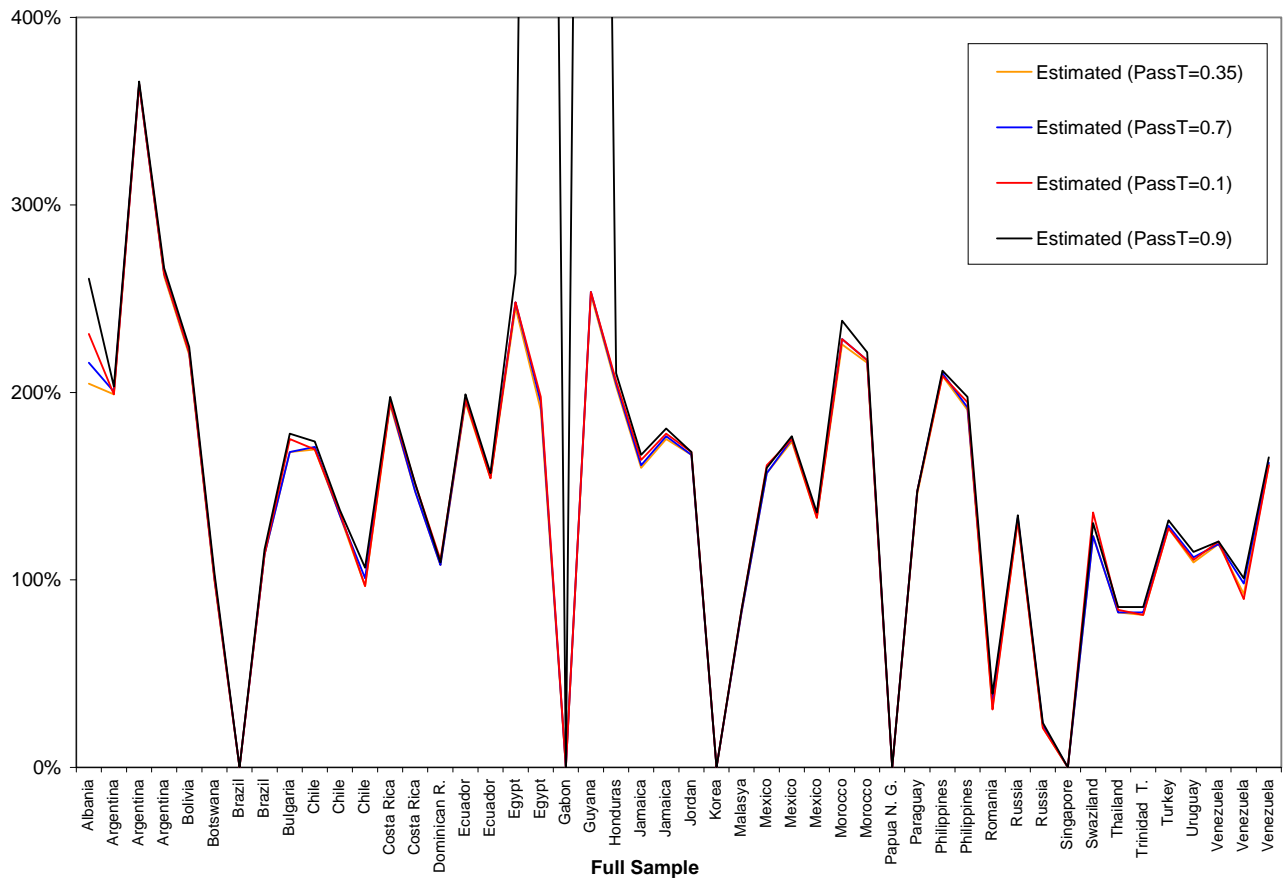


Figure 12: Devaluation Estimated and Pass-through

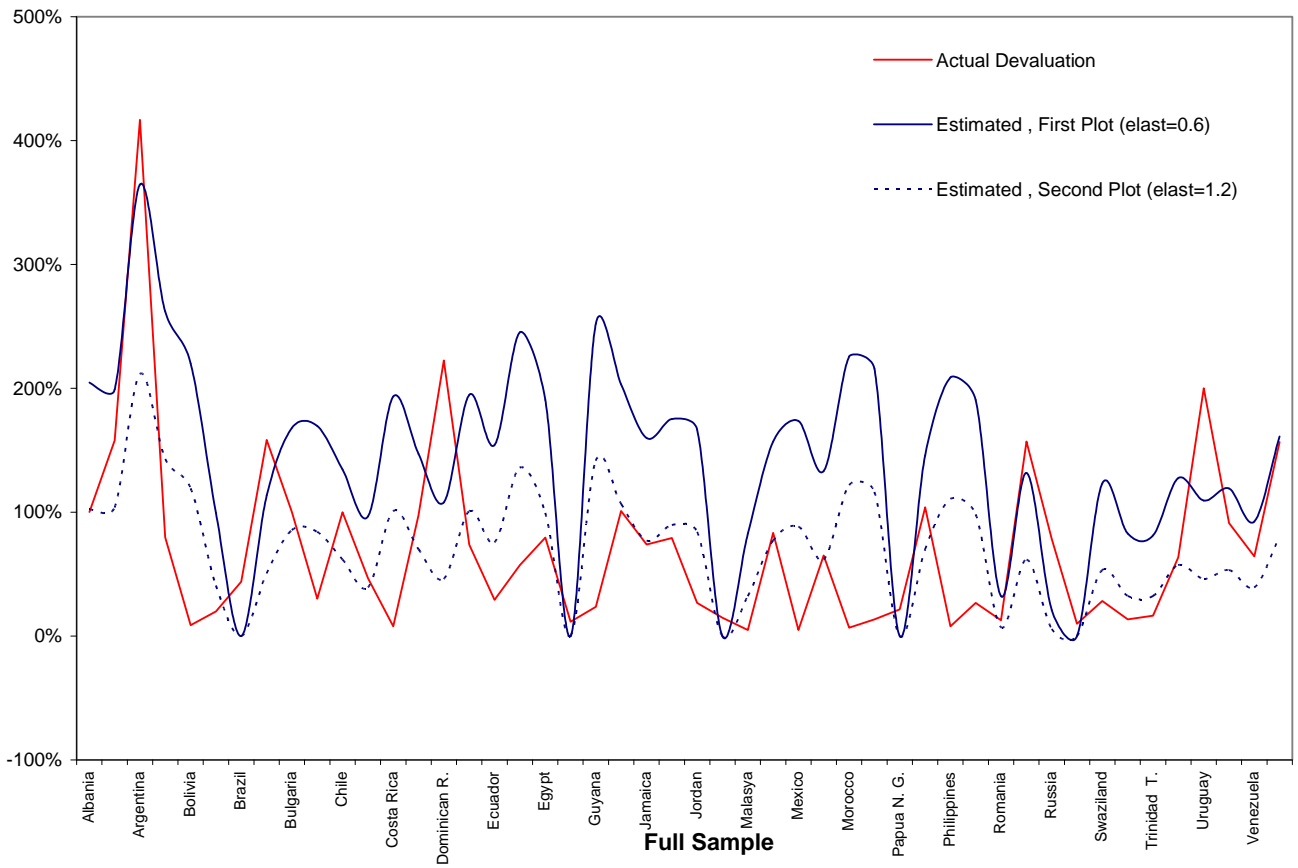


Figure 13: Actual Devaluation Versus Estimated Devaluation (Full Sample)

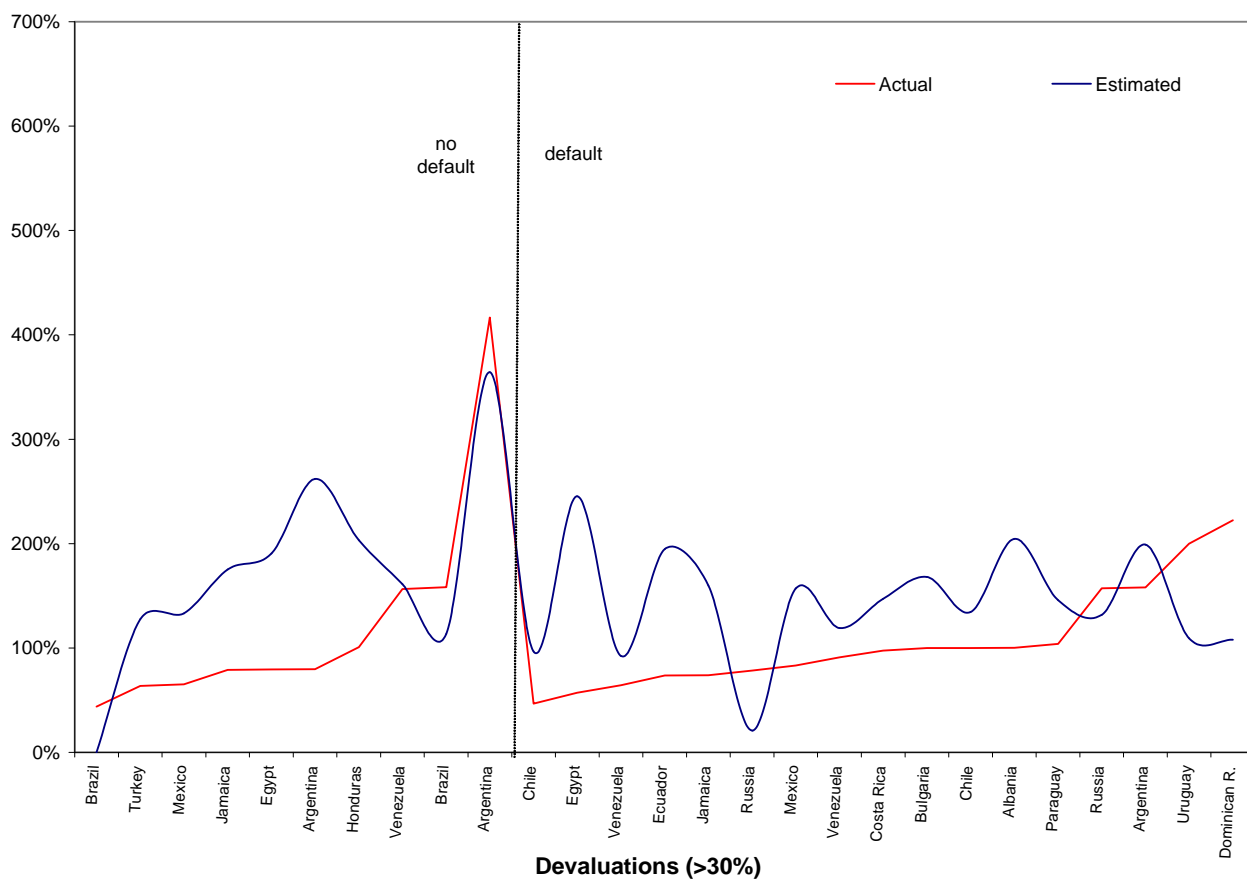


Figure 14: Actual Devaluation Versus Estimated Devaluation (Restricted Sample)

Table 1: Equilibrium Prices and Investment

$k^l(E(a_{t+1}))$	$q(Ez) =$	$q^*(Ez^*) =$
$k^n(1)$	β	β
$k^{\pi^* \pi} (\pi^* \pi \alpha^\phi + \pi^* (1 - \pi) \alpha + (1 - \pi^*))$	$\beta \cdot \{1 - \pi^* \pi [1 - \phi]\}$	$\beta \{1 - \pi^* [1 - \pi]\}$
$k^\phi(\alpha^\phi)$	$\beta \cdot \phi$	β
$k^d(\alpha)$	β	0

Table 2: Brazil(98)-Before Exchange Rate Devaluation

<i>Length of Public Debt (Years)</i>	<i>l</i>
<i>Debt Relative to GDP</i>	%
Total Public Debt - $B + B^*$	53
External (Public) Debt - B^*	23
Local Currency (Public) Debt - B	30
<i>Flow Relative to GDP</i>	%
Exports	7
Imports	7
Trade Balance - TB	0
Financial Cap.-Outflow - $B^*(1 - q^*)$	3
Investment - $k' - k(1 - \delta)$	16
Private Consumption - c	59
Public Expenditure - g	22
Interest Rate - Local Debt - $B(1 - q)$	2
Tax - θ	30
<i>Parameters</i>	-
β	0.9524
$f(k) = Ak^\lambda$ and δ	$10.k^{0.4}$ and 0.05
α	0.05
τ	0.35
α^ϕ	$[1 - \frac{\bar{a}}{1-\bar{a}} \bar{k} \ln(1 + \varkappa(\phi))^{(1-\bar{a})}]$
η	0.6
η^*	0.6
π	0.61
π^*	0.20

Sources: IMF, Central Bank, IPEA, Paiva (2003,[17]).

Table3 : Selected Countries

Country	Currency Crises Start of Episode(y/m)	Debt Default ¹ ? Start of Episode	Actual devaluation (%)	EXP/GDP (%)	IMP/GDP (%)	TAX/GDP (%)	B [*] /GDP (%)	B/GDP (%)	Free risk rate (%)	π^* (%)	π (%)	b_{ind} /GDP (%) ²	³		
Albania	1992	7	y	1992	100	11	89	21	67	0	5	15	30	25	y
Argentina	1990	12	n		80	10	5	10	43	17	6	6	87	92	y
Argentina	1989	11	n		417	13	7	10	83	40	3	66	95	90	y
Argentina	2001	12	y	2001	158	12	10	12	39	16	5	5	78	15	n
Bolivia	1989	7	y	1988	9	22	23	13	83	4	5	40	67	16	n
Botswana	1975	7	n		20	44	64	30	39	19	3	3	49	94	y
Brazil	1998	12	n		44	7	7	30	23	30	5	20	61	24	n
Brazil	1993	6	n		158	11	9	26	29	62	5	9	94	96	y
Bulgaria	1992	12	y	1992	100	47	53	36	114	115	12	55	65	90	y
Chile	1984	8	y	1985	30	24	25	29	69	25	5	31	75	26	n
Chile	1982	5	y	1983	47	19	21	29	35	4	8	30	60	26	n
Chile	1971	12	y	1972	100	11	12	15	25	21	8	37	67	12	n
Costa Rica	1987	8	y	1987	8	32	36	21	98	18	5	19	90	25	n
Costa Rica	1980	12	y	1981	98	26	37	17	48	28	8	7	33	82	y
Dominican R.	1984	12	y	1982	223	28	33	11	29	19	9	85	60	80	y
Ecuador	1982	4	y	1982	29	22	24	11	44	0	3	37	65	18	n
Ecuador	1999	9	y	1999	74	32	25	14	88	9	7	15	76	30	n
Egypt	1978	12	n		79	22	37	38	86	62	8	13	60	90	y
Egypt	1989	7	y	1987	57	18	32	29	112	74	7	8	93	8	n
Gabon	1980	9	y	1978	12	65	32	36	35	4	14	23	50	77	n
Guyana	1983	12	y	1982	24	46	65	43	255	193	6	16	60	10	n
Honduras	1990	2	n		101	29	34	15	93	16	8	4	84	89	y
Jamaica	1983	10	n		79	36	43	25	94	54	6	13	67	90	y
Jamaica	1991	8	y	1990	74	50	51	32	111	0	7	6	88	82	n
Jordan	1988	8	y	1989	27	45	67	14	98	52	3	16	25	73	y
Korea	1997	6	n		15	35	36	17	11	1	8	10	67	41	n
Malasya	1985	12	n		5	54	49	27	54	30	8	4	73	112	y
Mexico	1989	1	n		5	20	19	13	51	27	5	6	74	92	y
Mexico	1994	11	n		65	17	22	13	29	8	4	18	70	86	y
Mexico	1982	2	y	1982	83	10	13	14	27	29	9	3	87	11	n
Morocco	1984	11	y	1985	7	24	34	24	108	32	7	7	52	16	n
Morocco	1983	7	y	1983	13	21	30	25	92	35	5	15	61	16	n
Papua N. G.	1994	8	n		22	56	38	23	36	16	9	5	35	56	n
Paraguay	1989	2	y	1987	104	34	37	10	59	2	5	22	66	38	n
Philippines	1986	1	y	1986	8	24	22	12	78	16	3	25	54	20	n
Philippines	1983	9	y	1983	27	22	28	12	63	18	8	4	85	83	y
Romania	1996	1	n		13	28	33	30	20	24	8	6	83	26	n
Russia	1992	12	y	1991	78	62	48	16	31	14	8	72	75	35	n
Russia	1998	7	y	1998	157	31	25	20	57	29	5	60	62	26	n
Singapore	1997	7	n		10	131	139	40	17	59	8	3	66	52	n
Swaziland	1985	7	n		28	57	85	29	64	1	6	21	59	88	y
Thailand	1997	3	n		14	48	47	18	41	0	8	4	27	81	n
Trinidad T.	1988	7	y	1989	17	39	34	29	47	17	5	19	78	40	n
Turkey	2001	1	n		64	24	32	28	45	27	5	4	75	96	y
Uruguay	1982	11	y	1983	200	14	17	21	27	18	7	5	79	19	n
Venezuela	1989	2	n		157	20	27	19	51	5	4	3	67	92	y
Venezuela	1984	1	y	1982	64	20	11	22	36	0	3	19	58	33	n
Venezuela	1995	11	y	1995	91	27	22	16	43	24	8	25	79	23	n

To reach comparative results, we set scale factor (A) equals to 15 for all countries, including Brazil. 1: Default or restructuring of the external debt. Many episodes lasted several years. 2: b_{ind} is the debt for default-devaluation indifference, and its value split the crisis zone into two. 3: Only to compute the critical debt levels avoiding empty crises zones, in some countries, we consider that government has constant endowment of 0.75 GDP/year. (n) means no endowment and (y) means 0.75-endowment.

Sources: The World Debt Tables, World Bank Book ; International Financial Statistics, Yearbook / IMF ; World Development Indicators, The World Bank ; Country Info Base free available in <http://www.dbresearch.de> ; Central Banks.

2.6 Appendix: Effect of Real Devaluation on the Trade Balance

Defining Exp as exports measured in domestic output units, Imp as imports denominated in units of tradable, R_1 as the initial real exchange rate, and R_2 as its new level after devaluation, we can compute the trade balance change $D(\cdot)$ as:

$$\begin{aligned}
 TB(R) &= Exp(R) - Imp(R)R \\
 \frac{\Delta TB}{\Delta R} &= \frac{\Delta Exp}{\Delta R} - \frac{\Delta Imp}{\Delta R}R_2 - Imp(R_1) \\
 \frac{\Delta TB}{\Delta R} &= \frac{\Delta Exp}{\Delta R} \frac{R_1}{Exp(R_1)} \frac{Exp(R_1)}{R_1} - \frac{\Delta Imp}{\Delta R} \frac{R_1}{Imp(R_1)} R_2 \frac{Imp(R_1)}{R_1} - Imp(R_1) \\
 \frac{\Delta TB}{\Delta R} &= \left[\eta \left(\frac{Exp(R_1)}{R_1 \cdot Imp(R_1)} \right) + \eta^* \frac{R_2}{R_1} - 1 \right] Imp(R_1)
 \end{aligned}$$

Where $\eta = \frac{\Delta Exp}{\Delta R} \frac{R_1}{Exp(R_1)}$ and $\eta^* = -\frac{\Delta Imp}{\Delta R} \frac{R_1}{Imp(R_1)}$. Defining σ as the exports-imports ratio, $R_1 \equiv 1$, and $R_2 \equiv R$, we obtain

$$\Delta TB = (R - 1) [\eta\sigma + \eta^*R - 1] Imp(1)$$

3 Inflation Targeting, Credibility and Confidence Crises

Considering the inflation targeting regime, we study the interplay between central bank transparency, credibility, and the target level for inflation. Based on a model developed in the spirit of the global games literature, we argue that over-the-target inflation of the self-fulfilling type may arise whenever *a weak* central bank adopts *a high degree of transparency and a low target level*. The central bank is considered weak when good luck (exceptional favorable state of nature) is required to the target be achieved. On the other hand, if a weak central bank opts for less ambitious goals, namely lower degree of transparency and higher target level, it may avoid confidence crises and ensure a unique equilibrium for the expected inflation. Moreover, even after ruling out the possibility of confidence crises, less ambitious goals may be desirable in order to attain a higher credibility and hence a better expectation coordination.

3.1 Introduction

To manage an adequate inflation targeting regime, central banks must consider the trade-off between the *ideal* framework and a *more defensible* one. We argue that it is an important issue since confidence crisis³⁰ may arise when central bank adopts a too ambitious framework. By too ambitious we mean low target level and high degree of transparency when the credibility³¹ is not compatible.

In this way, less ambitious framework may prevent over-the-target inflation. Moreover, even if the economy is not subject to self fulfilling crisis, higher target and less transparency may be required in order to attain a higher central bank's credibility.

We first propose a basic model to appraise how the target level should be set in the presence of common uncertainty about the commitment strength. We name "common" because the uncertainty present in the central bank's office is the same as the uncertainty present in the private sector's office³². We conclude that a higher target for inflation increases the credibility in the precommitment making the optimal target higher than the one obtained using models where this increasing credibility effect is not considered, as in the Cukierman-Liviatan [5].

Second, extending the model to make confidence crises possible, multiple equilibria become possible too. In this case, there are three possible self-fulfilling expectations for inflation: the target level ("optimistic" equilibrium), the discretionary level ("pessimistic" equilibrium), and an inflation rate that is lower than the discretionary one but higher than the target ("not-extreme" equilibrium). As the target level becomes higher, the "not-extreme" equilibrium converges to the "pessimistic" one. On the other hand, if the target becomes high enough, multiple (bad) equilibria will be avoided and the "optimistic" one is ensured. The optimal target depends on the likelihood of each equilibrium to be selected and on the central bank's willingness to avoid a confidence crisis³³.

Third, we consider the possibility of "no-common" uncertainty about the commitment strength. In this case, the central bank and the public compute the same expected commitment-strength, but the public perceive a wider range for the commitment-strength realization. Our assumption is that the difference in ranges is decreasing in the degree of the central bank transparency. In addition, if central bank was fully transparent, the range difference would be zero and the uncertainty would be

³⁰Over-the-target inflation of the self-fulfilling type.

³¹Credibility is the extent to which agents believe that central bank will carry out its pre announced plan (the inflation target).

³²With such assumptions, transparency issue can not be addressed since the public (private agents) and the policymaker (central bank) have the same information set. The possibility of confidence crises is also neglected in this first approach. We first consider that uncertainty is given by the distribution of the cost of preventing inflation, which *drives* but *does not depend on* the expectations.

³³We avoid "equilibrium selection theory" in this paper. On the other hand, we plot the welfare for each equilibrium versus policy variables on the same figure. In this way, the reader can compute the best policy for 'max-min' policymaker, for example.

“common”. Results indicate that not only higher targets but also less transparency may help central bank in avoiding confidence crises.

Finally, when perturbing common-knowledge between private agents, uniqueness is ensured even considering speculative attacks, as in Morris-Shin[6]. The first result is also recovered, i.e. a higher target for inflation increases the credibility in the precommitment. Adding a *precise* public signal, self-fulfilling actions and equilibrium multiplicity may still exist even for a slightly lack of common knowledge between private agents (as in Angeletos and Werning[1]). In such case, as the target level becomes higher, the “not-extreme” equilibrium converges to the “pessimistic” one again. On the other hand, if the target becomes high enough, multiple (bad) equilibria will be avoided. The optimal target depends on the likelihood of each equilibrium to be selected and on the central bank’s willingness to avoid a confidence crisis. Again, results indicate that more precise public information may open the door to bad equilibrium, contrary to the conventional wisdom that more central bank transparency is always good when an inflation targeting regime is considered.

3.2 Basic Model

The framework used as a starting point to our analyze is based on the Cukierman-Liviatan model [5] and it is similar to the one presented in Barro and Gordon [2]. It has some features that consider the agent’s uncertainty about the commitment enforcement and suggest that the optimal target should be decreasing in the central bank credibility. On the other hand, the Cukierman-Liviatan model considers a naive uncertainty approach, based on two central bank types: the “strong” one which always adheres to the announced policy and the “weak” one which does it only as an *ex-post* expedient. We next purpose some extensions. First, their model does not encompass the possibility of a more sophisticated central bank’s decision, namely: to fulfill the target depending on the intensity of an occasional shock observed after the target announcement. Second, the commitment strength may depend on the credibility, and the credibility itself should affect the central bank’s decision about respecting (or not) its commitment. Finally, no-common uncertainty about the commitment strength³⁴, as well as public strategic behavior should be considered. We next describe the Cukierman-Liviatan original model and further extend it to address the previous comments.

There are two types of agents: the central bank and private agents (public). Actions are taken in three stages: the central bank announces the target for inflation (π_a), expectations are formed (π_e) by the public and actual inflation is chosen (π). There are two central bank types $i \in \{1, 2\}$ with different abilities to precommit. The first type (“strong”) always fulfills its commitment while the second one (“weak”) does it only if it is *ex post* expedient. Their objective function is positively related to surprise inflation and negatively related to actual inflation, as follows³⁵:

$$\begin{aligned} v^i(\pi_e, \pi_a) &= \max_{\pi \geq 0} A[\pi - \pi_e] - \frac{\pi^2}{2} - c^i(\pi_a, \pi) \text{ where } c^i(\pi_a, \pi) = \begin{cases} k^i, & \pi_a \neq \pi \\ 0, & \pi_a = \pi \end{cases}, i \in \{1, 2\} \\ k^1 &= A^2, k^2 = 0, A > 0, \pi_a \geq 0, \text{ and } \pi_e \geq 0 \end{aligned}$$

Note that the central bank best response for actual inflation (π^*) is either the target level (π_a) pre-announced or the discretionary inflation level (A). We can compute the welfare gain (w_a^i) of type (i) keeping the target (π_a) as follows³⁶:

$$w_a^i = k^i + f(A, \pi_a), \text{ where } f(A, \pi_a) \equiv A(\pi_a - A) - \frac{\pi_a^2}{2} + \frac{A^2}{2}$$

³⁴Each private agent and the central bank perceive different signals related to the commitment strength.

³⁵We add the “cost of not fulfilling the target” function $c^i(\pi_a, \pi)$ to the original Cukierman-Liviatan model [5] to formalize that the strong type always fulfills the pre-announced target while the weak type is not concerned about the previous announcement.

³⁶From now on, the welfare gain from keeping actual inflation on the target will be denoted by (w_a).

Since the goal of the inflation target is to coordinate expectations from the discretionary inflation level (A) to socially optimal level (0), it is easy to check that ($w_a^1 > 0$) and ($w_a^2 \leq 0$) for any $\pi_a \in [0, A]$, and both types of ability are justified for any possible target level.

There is a continuum of private agents without a strategic behavior. Their role is to process information, to form beliefs concerning the central bank's type and to compute the expected inflation. First, it is assumed that the private expectation about the central bank type is formed based on the exogenous probability (α) of the type being strong ($i = 1$), which is the same for all private agents. This probability measures the central bank's credibility. The expected inflation is given by:

$$E[\pi|\alpha, \pi_a] = \pi_e = \alpha\pi_a + (1 - \alpha)A$$

Based on this framework Cukierman and Liviatan [5] answered the following question: “*what should be the optimal announcement π_a^* for each type (i)?*”. For ($\alpha = 1$), the target and the expected inflation are the same. Then, the central bank type 1 promises and delivers zero inflation rate. If we consider $\alpha \in (0, 1)$, the central bank 1 promises and delivers $A(1 - \alpha)$ inflation rate. As (α) tends to zero the announcement effect on the expectations vanishes and the central bank 1, who always keeps its promises tends to pre announce the discretionary inflation level rate. Although type 2 ends up inflating at the discretionary rate, it has an interest to keep itself indistinguishable at the announcement stage in order to stimulate lower expectations ($\pi_e < A$). It follows that $\pi_a^{*i} = A(1 - \alpha)$ for both types (i). Accordingly, full credibility ($\alpha = 1$) is not required for inflation targeting to be implemented. In the absence of pre commitment, the result leads to an inflationary bias (A) that can be reduced whenever central banks are able to precommit with some credibility ($\alpha > 0$). This bias reduction improves welfare. To totally eliminate the inflationary bias and to achieve the socially optimal inflation rate (zero), the ability to commit must not be only present but must also be undoubtedly recognized by the public. Otherwise, a lower inflationary bias reappears.

Next, we gradually extend this framework to argue that there are some other reasons for the inflation target to be higher than the socially optimal level.

3.3 Endogenous Credibility

Considering the endogenous credibility we now compute not only the effect of the credibility on the target, but also the effect of the target choice on the credibility.

Less ambitious (*higher*) targets are attained more often when monetary policy is subordinated to fiscal financing requirements and the economy is subject to shocks that can make more inflation tolerable. In this sense, (α) should not be an exogenous variable because when (π_a) is selected (α) should be affected. Therefore, we consider the uncertainty about the ability to precommit coming not from some private suspicion related to the central bank type. Perhaps, the cost of being above the target varies as from the announcement stage if the economy is hit by an adverse shock and *the uncertainty about the future inflation may be present in the central bank's office too*.

The model considered here is the same as the previous one, but with only one type of central bank, which is common knowledge. Instead of being a real number, the cost of not fulfilling the target (k) is now uniformly distributed³⁷ on the support $[\underline{K}, \overline{K}]$ and it is drawn after the public's expectations have been formed. Actual inflation is chosen at the end of the period. A low realization of k can be viewed as a shock that decreases the value of keeping the commitment without using inflation short run effects. If we set ($\overline{K} = 0$) or ($\underline{K} = A^2$) the equilibria can be computed as follows: with ($\alpha^* = 0$) and discretionary inflation rate, or with ($\alpha^* = 1$) and zero inflation rate, respectively. To keep attention on the intermediate case where $\alpha^* \in (0, 1)$, we assume that k is drawn from $U[0, B]$, with $B > 0$ ³⁸.

³⁷Further, we also propose a similar framework based on uncertainty described with normal distribution.

³⁸ $\underline{K} = 0$ and $\overline{K} = B > 0$.

Depending on the values of k and π_a the commitment is delivered or not. The credibility is given by:

$$\begin{aligned}\alpha(\pi_a) &= \text{prob}(w_a(k, \pi_a) > 0) \\ \alpha(\pi_a) &= \max \left\{ 1 - \frac{1}{B} \left[A(A - \pi_a) - \frac{A^2}{2} + \frac{\pi_a^2}{2} \right]; 0 \right\}\end{aligned}$$

When choosing the target, the central bank understands that the higher is its level, the more credible its policy tends to be. In particular, only A -inflation commitment is fully credible.

As in the previous model, because the possibility of the cost of not fulfilling the target being positive, the commitment is heard. Thus, commitment drives expectations and adds value to the economy. But now we have a different answer to the following question: “*what should be the optimal target π_a^* ?*”. On the one hand, for a given (α) , the closer to zero the target announcement is, the lesser is the expected inflation since (π_a) drives it. This fact increases welfare to any fixed $(\alpha \neq 0)$. But on the other hand, the closer the target announcement to zero is, the closer to the zero (or equal) the credibility (α) is.

With this background in mind we define the economy with only one type of central bank; given by the parameter (A) and the common knowledge distribution of $k \sim U [0, B]$; with two positive parameters (A, B) . The following proposition characterizes the equilibrium:

Proposition 1 *For any economy $(A > 0, B > 0)$, the equilibrium target (π_a^*) exists, it is unique, and it is in the interior of the set $[0, A]$. If we use the endogenous credibility obtained $\alpha(\pi_a^*)$ to solve the original Cukierman-Liviatan model, we will obtain a new optimal target (π_a^{**}) , which will be always lower than (π_a^*) . Proof: Appendix.*

A less ambitious target improves credibility in the announcement and induces positive welfare effect for economies where the central bank is not able to set “fully-credible” commitment. Then, when setting targets, the central bank must be aware that the announcement effect on expectations is reduced by credibility **and** that the announcement itself affects credibility. We present in the Figure 1 the optimal target announced when considering the credibility effect versus the one announced when this effect is neglected. Note that, the weaker the ability in pre commitment (lesser B) is, the higher the difference between the two announcement values is³⁹.

3.3.1 Self-fulfilling Inflation

One possible reason to assume some cost for not fulfilling the target may come from the fact that the public may use the central bank’s decisions to learn and to compute next expectations. In this sense, to fail to fulfill the target may be “punished” by credibility loss in the subsequent target announcement. With such assumptions, credibility (α) may also affect the central bank’s incentive to defend the target. The value of respecting the target may be increasing in the current credibility whenever benefits from “keeping the target decisions” are computed in a much slower way than the credibility loss associated with “not keeping the target decisions”. This feature may open the door for confidence crises and self-fulfilling inflations.

Keeping the three-stage framework, we define the economy (ξ) with the following set of parameters $\{(A > 0); (\pi_a \in [0, A]); (\epsilon \geq 0); (n \in \mathbb{R})\}$ plus the increasing function $h(\alpha)$, which are all common knowledge. The central bank type is unique and is the same as in the first-proposition model except for the cost of “not keeping the target” function, now defined as follows: $c(\pi \neq \pi_a) \equiv h(\alpha) + k$ and zero if $(\pi = \pi_a)$. k is a random variable distributed according to $U[n - \epsilon, n]$, (α) is the endogenous credibility that solves $\alpha = \text{prob}(\pi = \pi_a | \alpha)$, and $h(\cdot)$ ⁴⁰ measures how much the cost of

³⁹On the other hand, as (B) increases the credibility (α^*) becomes closer to one and the difference between π_a^* and π_a^{**} is smaller.

⁴⁰In order to reach a simple characterization of the equilibrium we also assume that $h : [0, 1] \rightarrow \mathbb{R}$ is linear.

not keeping the target depends on the public expectations. The timing of actions is the same: the target is announced, expectations are formed, uncertainty (k) is solved and actual inflation is implemented. With such assumptions, both fundamental and expectations shocks may be important to compute the central bank's incentives in the choosing actual inflation, since the welfare gain (w_a) from keeping the target can be computed as follows:

$$w_a(k, \alpha) = k - x(\pi_a) + h(\alpha)$$

$$x(\pi_a) \equiv A(A - \pi_a) + \frac{\pi_a^2}{2} - \frac{A^2}{2}$$

It is always possible to reach an equilibrium for any economy (ξ) and it may be possible to reach more than one. When the uncertainty about the future central bank's incentives is high, i.e. ($\epsilon > h(1) - h(0)$), we classify the economy (ξ) as (ξ^u) type economy, otherwise we classify the economy (ξ) as (ξ^m) type economy⁴¹. The following proposition characterizes the equilibrium:

Proposition 2 *The economy ξ always admits an equilibrium. For ξ^u type economy it is possible that: (i) $x \leq (n - \epsilon + h(1))$ and only perfect commitment ($\alpha = 1$) equilibrium is possible, (ii) $x \in (n - \epsilon + h(1), n + h(0))$ and only imperfect commitment ($\alpha \in (0, 1)$) is possible, (iii) $x \geq n + h(0)$ and only discretionary ($\alpha = 0$) equilibria is possible. For (ξ^m) type economy it is possible that: (i) $x \leq (n - \epsilon + h(1))$ and perfect commitment equilibrium is possible, (ii) $x \in (n + h(0), n - \epsilon + h(1))$ and perfect commitment, imperfect commitment and discretionary equilibria are possible, (iii) $x \geq n + h(0)$, and discretionary equilibria is possible. Proof: Appendix.*

According to this proposition, if there is too much uncertainty concerning future central bank's incentives ($\epsilon > h(1) - h(0)$), then the equilibrium is unique: perfect commitment for very strong central bank, discretionary for very weak central bank and imperfect commitment otherwise. When ϵ is sufficiently large, there is no room for self-fulfilling inflation.

On the other hand, if the region for the possible central bank's incentives shrinks (ϵ decreases), the uniqueness remains only for a very strong or a very weak central bank. The intuition is that some economies may be subject to multiple equilibria when the decision about respecting the target or not depends much on the credibility (α) before the k 's assortment. In such case, to relax (to increase) the target for inflation may have two welfare effects. First, and the new one, it is possible that only perfect commitment equilibrium remains when the target is increased. Second, as long as $h'(\cdot) > \epsilon$, the *critical* k^{*42} becomes greater when the target is increased, and hence the state region for good expectations shrinks. Then, the announcement credibility may be increasing in the target or not, whenever ($\epsilon < h(1) - h(0)$).

As we have shown in the Figure 2, when $h(\alpha) \equiv \rho\alpha$ and multiple equilibria are possible, to increase the target *may* be a good deal if it avoids multiplicity. But this decision also depends on the central bank's willingness to avoid a confidence crisis and on the probability of each equilibrium to be selected over the multiple equilibria region. Because of the coordination failure, any of them could be the one and, unfortunately, this model can not help us to compute their likelihood. Such difficulty is usually avoided by the definition of an arbitrary sunspot variable⁴³ that would allow us to compute expected welfare for each target. Obviously, the policy recommendations would be very different depending on the assumptions about the sunspot distribution and depending on the central bank's willingness to avoid crises.

An alternative public policy to avoid multiplicity may be related to the availability of the public information. In the numerical exercise presented in the Figure 3, we consider that central bank

⁴¹ u for unique and m for multiple equilibria.

⁴² k^* solves: $k = x - h\left(\frac{n-k}{\epsilon}\right)$.

⁴³ See Cole and Kehoe [4] for coordination failure in the public debt market.

can produce some noise (η) in the private information being less transparent. In this case, the k -distribution perceived by the central bank is uniformly distributed on $[n - \epsilon, n]$, but the k -distribution perceived by the public is uniformly distributed on $[n - \epsilon - \eta, n + \eta]$, with ($\eta \geq 0$). Results show that less transparency may avoid confidence crises when the target level considered is equal to 3%.

In the following section, we consider the public as a strategic player to appraise the information issue in a more sophisticated way. Up to this point public's role has been to process information, to form beliefs concerning the central bank's incentive and to compute inflationary expectations. Adding strategies and payoff structure to private agents and assuming an exogenous information structure, we can appraise the coordination aspect in a different way. The coordination motive arises from the strategic complementarity in public actions. Moreover, since the mass of speculators is decreasing in the endogenous credibility, next section also provides one possible interpretation for the previous function $h(\alpha)$.

3.4 Self-Fulfilling Inflation with Imperfect Information

The economy (ξ) is defined as a one-shot game with two stages, the function $h(\alpha) = \alpha\rho$ and the following set of parameters $\{(A > 0), (\rho > 0), (\pi_a \in [0, A]), (c > 0), (\sigma > 0), (\sigma_p > 0)\}$. The last two of those define information structure as we describe next. In the last stage of the game the central bank chooses the actual inflation after observing the speculative actions ($1 - \alpha$). The central bank keeps the inflation equal to the target (π_a) if and only if ($w_a \equiv k + \rho\alpha - x(\pi_a) \geq 0$)⁴⁴. Otherwise it inflates at level A . k is drawn in the beginning of the game from the support of the improper uniform (over the entire real line), but its value is not observed directly by the public (speculators).

The population of speculators is continuous and normalized to unit. Each speculator (j) may set α^j equal to one or zero. If he sets α^j equal to zero he believes that the target will probably⁴⁵ not be reached. With some cost, he speculates based on his beliefs (buying foreign currency, for example). If he sets α^j equal to one he believes that the target will probably be reached. In this case, he does not bet against the central bank (keeping savings denominated in local currency, for example). Then, the size of the attack ($1 - \alpha$) is given by $(1 - \text{prob}(\alpha^j = 1))$. Each (j) payoff is defined as being equal to $(1 - \alpha^j)(g_s - c)$. The speculative gain g_s depends on the central bank's response. If the target is sustained, then $g_s = g_a$, otherwise $g_s = g_A$, where ($g_A > c > g_a$). With this payoff structure, to speculate is a good deal only when the target is abandoned since ($g_A - c > 0$) and ($g_a - c < 0$). The incentive to attack tends to be increasing in the size of the attack. As we have argued, in some economies, the target may be abandoned when credibility is low. Note that, the greater the size of the attack is, the lower the credibility is.

To keep our framework as close as possible to the one proposed in Angeletos and Werning [1] we define $g_A \equiv 1$ and $g_a \equiv 0$ and consider that the strength of the status-quo k is not common knowledge. Instead of observing the realization of the k -value, each player (j) observes the public signal (s^p) and the private signal (s^j),

$$\begin{aligned} s^j &= k + \sigma\varepsilon_j; \sigma > 0 \text{ and } \varepsilon_j \sim N(0, 1) \\ s^p &= k + \sigma_p\varepsilon_p; \sigma_p > 0 \text{ and } \varepsilon_p \sim N(0, 1) \end{aligned}$$

(ε_j) is assumed to be independent of (k) and ($\varepsilon_{j'}$) for all $j' \neq j$. (ε_p) is also assumed to be independent of (k) and (ε_j). $N(0, 1)$ denotes the standard normal distribution.

⁴⁴ x is given by $A(A - \pi_a) + \frac{\pi_a^2}{2} - \frac{A^2}{2}$ and the no-attack-mass defined by (α) is increasing in the aggregate credibility: $\text{prob}(\pi = \pi_a | \text{public information})$

⁴⁵With probability higher than (c).

3.4.1 The Equilibrium

Results are based on monotone equilibria defined as perfect Bayesian. For each public signal, the agent (j) attacks if and only if its private signal (s^j) is less than some threshold $s^*(s^p)$. The mass of agents that ends up attacking is given by:

$$prob(s^j < s^*(s^p, \pi_a) | s^p, k, \pi_a) = \Phi\left(\frac{s^*(s^p, \pi_a) - k}{\sigma}\right) = 1 - \alpha$$

where $\Phi(\cdot)$ denotes the cumulative distribution function for the standard normal. The central bank will sustain the target if and only if k is greater than k^* , which is given by:

$$k^*(s^p, \pi_a) = x(\pi_a) + \rho \cdot \Phi\left(\frac{s^*(s^p, \pi_a) - k^*(s^p, \pi_a)}{\sigma}\right) - \rho$$

The expected payoff from attacking must be equal to zero whenever $s^j = s^*(s^p, \pi_a)$ ⁴⁶, which implies the following indifference condition:

$$\sqrt{\tau} \cdot \Phi^{-1}(c) = k^*(s^p, \pi_a) - \frac{\tau s^*(s^p, \pi_a)}{\sigma^2} - \frac{\tau s^p}{\sigma_p^2}, \text{ where } \tau = \frac{\sigma_p^2 \sigma^2}{\sigma_p^2 + \sigma^2}$$

Which after replacing $s^*(k^*)$ becomes:

$$\Phi^{-1}\left(\frac{k^* + \rho - x(\pi_a)}{\rho}\right) = \frac{\sigma}{\sigma_p^2} [k^* - s^p] + \frac{\sigma}{\sqrt{\tau}} \Phi^{-1}(1 - c)$$

It is always possible to find at least one $k^* \in [x - \rho, x]$ that solves this equation and this solution will be unique for every public signal (s^p) if and only if $\sigma \in (0, \frac{\sigma_p^2 \sqrt{2\pi}}{\rho}]$.

For any (positive) doubt related to the public signal, σ_p , uniqueness is ensured by a sufficiently small (positive) doubt related to the private signal, σ . That is the first proposition from Angeletos-Werning [1] and states that the multiplicity may vanish when the common knowledge is perturbed, as in Morris and Shin [6]. This result always holds for some exogenous information structure because precise private information anchors individual behavior and makes it difficult to predict the actions of others. Under the reasonable assumption that the improvement in the private signal implies improvement in the public signal, it is possible that public information becomes more precise faster than the private one, and so multiplicity may still exist even for small common knowledge perturbation ($\sigma \rightarrow 0$). In this case, the public signal drives individual behavior more than the private signal, motivating mass movements.

Keeping the exogenous information structure it is possible to set multiple-equilibria economies (ξ^m) assuming that $\sigma > \frac{\sigma_p^2 \sqrt{2\pi}}{\rho}$ and unique-equilibrium economies (ξ^u) assuming that $\sigma \in (0, \frac{\sigma_p^2 \sqrt{2\pi}}{\rho}]$.

Proposition 3 For (ξ^u) type economy, higher target increases the commitment credibility. For (ξ^m) type economy, higher target may turn the commitment more credible or not. The effect on the credibility will depend on the likelihood of each equilibrium $k^*(s^p)$ to be selected. Proof: Appendix.

The key intuition is that when the target is increased two effects are observed. First, the shock required for the commitment to be abandoned becomes greater (smaller k -realization), for any fixed ($\alpha \in (0, 1)$). This fact inhibits attacks and adds credibility. Second, as the central bank sets a higher target for inflation, new attack-strategies (or beliefs) are settled and this fact may increase the attack mass (decrease the credibility)⁴⁷. In this case, a higher target gives more room for over-the-target inflation. The first effect is always preponderant for (ξ^u) type economy.

⁴⁶ $s^*(s^p, \pi_a) = \sigma \cdot \Phi^{-1}\left(\frac{k^*(s^p, \pi_a) + \rho - x}{\rho}\right) + k^*(s^p, \pi_a)$

⁴⁷ $\alpha^*(\pi_a)$ may be decreasing in π_a .

For (ξ^m) type economy, the first effect tends to be preponderant when extreme equilibria are selected (the $k^*(s^p)$ closest to x (or to $x - \rho$)). Note that, when strategies are too optimistic or too pessimistic, the size of an attack is closer to zero or to one, respectively. For more pondered strategies, based on the not-extreme equilibrium $k^*(s^p)$, the attack-mass and the no-attack-mass are both significant. Then, enlargement in the attack size induced by more aggressive strategies is more relevant and critical k becomes greater for higher targets (see Figure 4). If not-extreme equilibrium tends to be the selected one in (ξ^m) type economy, to relax the target in order to get more credibility is a good idea only if multiplicity is avoided in many states (s^p) . Otherwise, the result would be more strength to speculative movement and the commitment enforcement would be reduced. In the Figure 5 we show extremes k -equilibria as a function of the public signal realization (s^p) . The parameters used are the same as those used in the Figure 4 and the target level considered in the benchmark case (black line) is equal to 2%. Note that either a higher target level (pink line) or a less transparency level (red line) may be used by central bank to eliminate multiple equilibria region.

3.4.2 Central Bank Transparency and Welfare Analyses

A lower (σ_p) value may be viewed as more central bank transparency. According to our results, more precise public information may open the door to bad equilibrium, contrary to the conventional wisdom that more transparency is always good in an inflation targeting framework. Some other papers have argued in the same direction, but based on different models. In Metz [3], more precise public information increases the likelihood of currency crises in case of bad fundamentals. Morris and Shin ([7] and [8]) have pointed out that welfare effect of increased public disclosures is ambiguous and that there is a dilemma between managing market prices and learning from market prices. They also conclude that when a Central Bank cannot actually control inflation⁴⁸, the inflation targeting regime could fail and undermine credibility. In this sense, it would be better for the central bank to simply forecast inflation and point out the extent to which its forecasts are contingent on fiscal policy. Our results suggest that inflation targeting may be a good set-up whenever the central bank can actually control *some* level of inflation in *some* states of nature.

In the Figure 6 we present the combined welfare⁴⁹ effect of a higher target and less transparency, considering that central bank knows its commitment strength in the beginning of the game, which is given by $(k_o = 3\%)$. We plot in the vertical axis the expected welfare cost for inflation only for the extreme equilibria cases. Note that as central bank becomes less transparent the welfare associated with the optimistic equilibrium is reduced, but the welfare associated with the pessimistic equilibrium is increased. Results also indicate that less transparency may be welfare improving out of the multiplicity region. Finally, for a lower target level (blue line) this welfare effect is present over a wider “unique-equilibrium-region”, but on the other hand, it is flatter than the effect observed under a higher inflation target level (red line).

⁴⁸Sargent and Wallace [9] is one of the many papers that have pointed out the limitations of the central bank's control over inflation.

⁴⁹For any given $\pi_a \in [0, A]$, the expected welfare is computed in the following way:

$$E \max_{\pi(s^p)} \left[A (\pi(s^p) - \pi_e(s^p)) - \frac{\pi(s^p)^2}{2} - c(\pi; \pi_a, s^p) \right]$$

$$\pi^e \equiv \Phi(s^{p*}) \cdot A + [1 - \Phi(s^{p*})] \cdot \pi_a$$

$$c(\pi; \pi_a) = \begin{cases} k + \rho \cdot (\alpha^*(s^p)) & ; \pi \neq \pi_a \\ 0 & ; \text{otherwise.} \end{cases}$$

and all uncertainty is given by (s^p) .

3.5 Concluding Remarks

Using different theoretical approaches, we appraise how the target level for inflation should be set in the presence of uncertainty about the ability in precommitting.

First, ruling out confidence crises, we conclude that higher target for inflation increases the credibility in the precommitment. This effect makes optimal target bigger than the target level selected when considering the same economy, except for the fact that the credibility effect is not computed⁵⁰.

When confidence crises are possible, multiple equilibria may arise. In this case, to set higher targets for inflation may stimulate over-the-target inflation and reduce the central bank credibility. On the other hand, multiple (bad) equilibria may be avoided. The optimal target will depend on the likelihood of each equilibrium to be selected and on the central bank's willingness to avoid a confidence crisis.

We restore uniqueness breaking common knowledge with exogenous information structure, as in Morris and Shin [6]. In this case, it is possible to ensure again that a higher target for inflation increases the credibility in the precommitment.

Finally, adding a *precise* public signal, confidence crises and equilibrium multiplicity may still exist even for a small lack of common knowledge, as in Angeletos and Werning ([1]). Precise public information may open the door to bad equilibrium, contrary to the conventional wisdom that more central bank transparency is always good news when considering the inflation targeting regime. In multiple equilibria case, to set higher targets for inflation may stimulate over-the-target inflation and reduce the central bank credibility. On the other hand, multiple (bad) equilibria may be avoided. Depending on the characterization of the *target strength* uncertainty, it may be optimal to have an *ideal* status-quo (socially optimal target for inflation under fully transparency) or a *more defensible* one (higher target and less central bank transparency). The optimal policy will also depend on the central bank's willingness to avoid confidence crises.

⁵⁰As in Cukierman and Liviatan ([5]).

References

- [1] Angeletos, G. ; Werning, I. (Forthcoming): “Crises and Prices: information aggregation, multiplicity and volatility”. *The American Economic Review*, 96.
- [2] Barro, R.; Gordon, D.(1983):“Rules, Discretion and Reputation in a Model of Monetary Policy”, NBER Working Paper Series, n° 1079.
- [3] Metz, C. (2002): “Private and Public Information in Self-fulfilling Currency Crises”, *Journal of Economics*, 76: 65-85.
- [4] Cole, H.; Kehoe, T. (1996): “A Self-Fulfilling Model of Mexico’s 1994-1995 Debt Crisis”, *Journal of International Economics*, 41:309-330.
- [5] Cukierman, A.; Liviatan, N. (1991): “Optimal accommodation by strong policymakers under incomplete information”, *Journal of Monetary Economics*, 27:99-127.
- [6] Morris, S. ; Shin, H. (1998): “Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks”, *American Economic Review*, 88(3): 587-597.
- [7] Morris, S. ; Shin, H. (2002): “Social Value of Public Information”, *American Economic Review*, 92(5): 1521-1534.
- [8] Morris, S. ; Shin, H. (2005): “Central Bank Transparency and the Signal Value of Prices”, *Brookings Paper on Economic Activity*.
- [9] Sargent, T. ; Wallace, N.(1981): “Some Unpleasant Monetarist Arithmetic”, *FED-Minneapolis, Quarterly Review*.

3.6 Figures

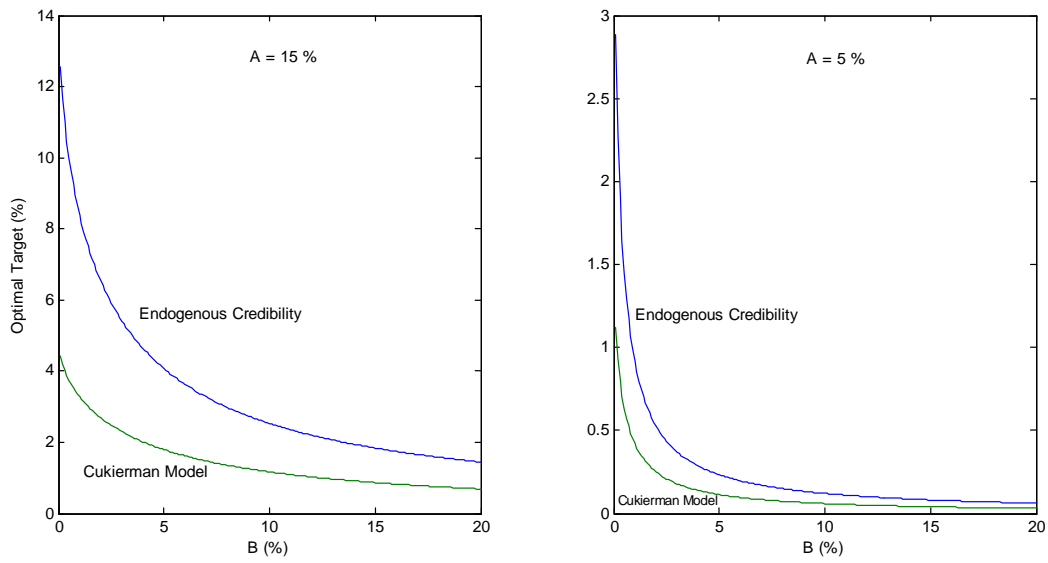


Figure 1: Optimal Targets

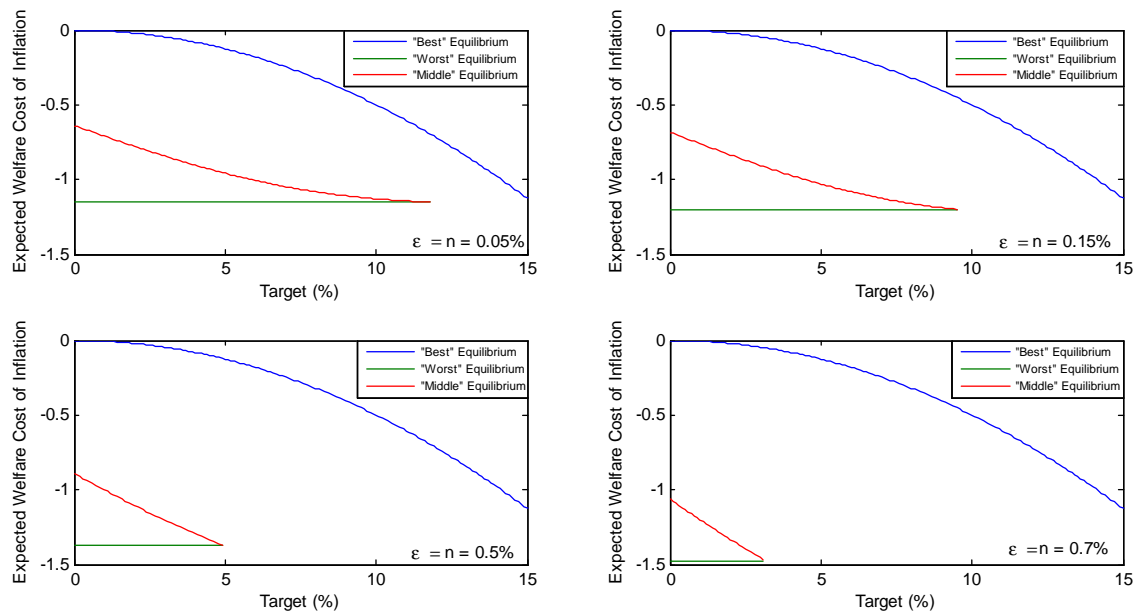
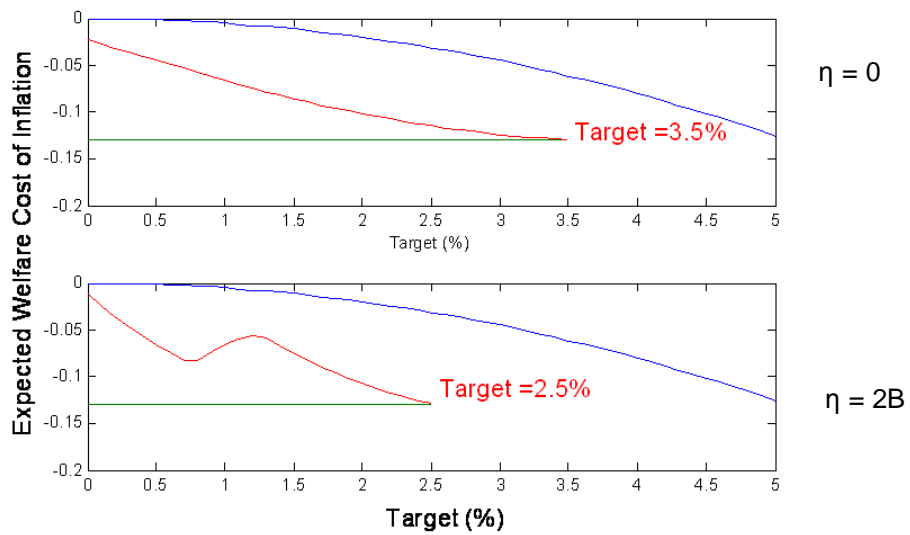


Figure 2: Self-fulfilling Equilibria ($A = 15\%$; $\rho = 2.5\%$)



$\rho = .15\%$; $A = 5\%$; $n = \varepsilon = B = .1\%$ for both figures.

Figure 3: Multiplicity and Transparency

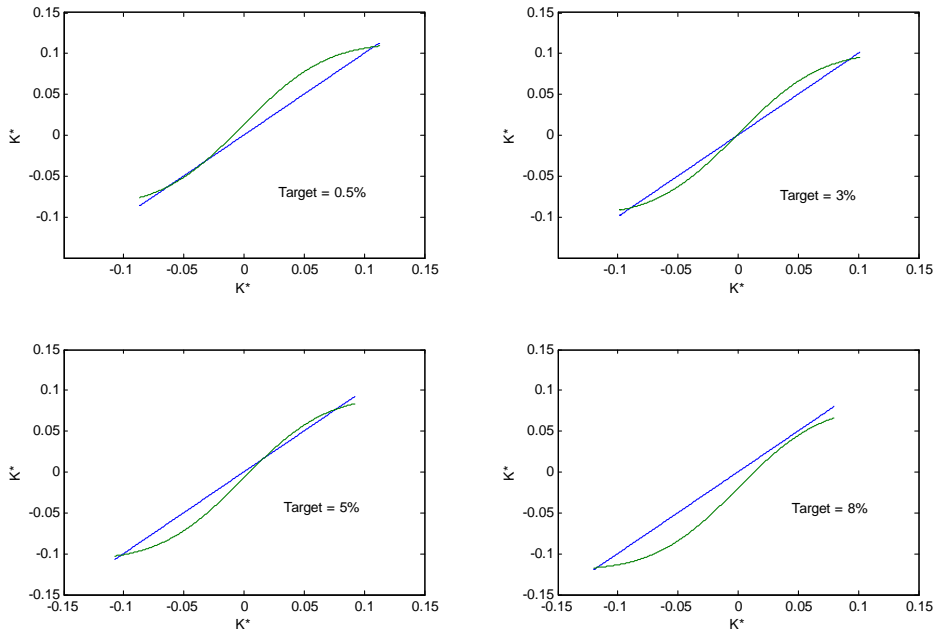


Figure 4: No-Common Knowledge K^* -Equilibrium
 ($s^p = 0, A = .5, \frac{\sigma}{\sigma_p} = \frac{.60}{.18}, c = .5, \rho = .2$)

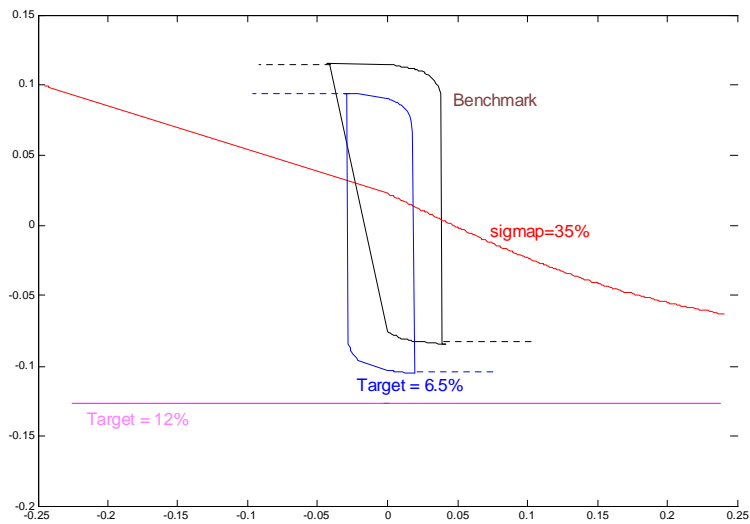


Figure 5: No common Knowledge Equilibrium as function of s^p
 Vertical axis: Critical k^* . Horizontal axis: Public signal (s^p)
 (Benchmark: Figure4's parameters are considered, $\pi_a=.02$)

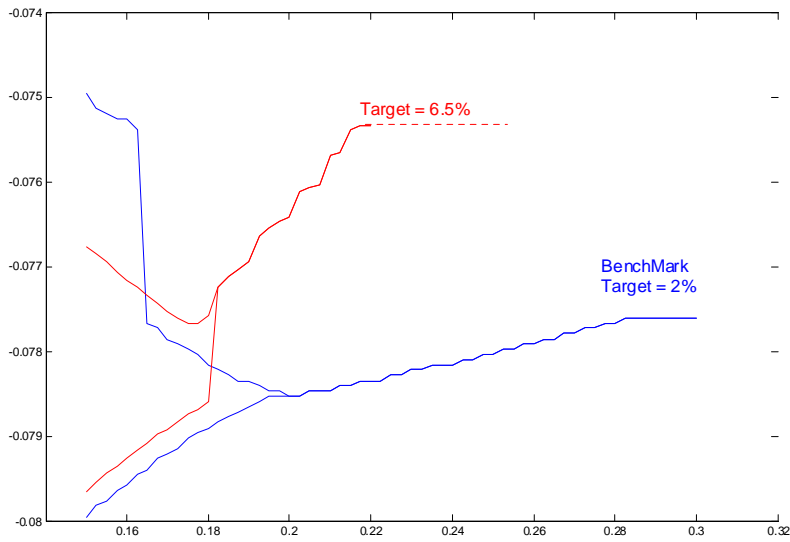


Figure 6: Welfare Cost in the Extreme Equilibria and Transparency
 Vertical axis: Inflation welfare expected cost. Horizontal axis: Standard deviation of the public signal
 (The Figure4's parameters are considered)

3.7 Appendix

Proof. Proof of proposition 1: The central bank from economy (A, B) solves the following problem:

$$\begin{aligned}\pi_a^* &= \arg \max_{\pi_a \in [0, A]} E [v(\pi_a, k)]; k \sim U [0, B] \\ v(\pi_a, k) &= \max_{\pi \geq 0} \left[A(\pi - \pi_e(\pi_a)) - \frac{\pi^2}{2} - c(\pi_a, \pi, k) \right] \\ c(\pi_a, \pi, k) &= \begin{cases} 0 & \text{if } \pi_a = \pi \\ k & \text{if } \pi_a \neq \pi \end{cases}\end{aligned}$$

and it is easy to check that,

$$\begin{aligned}\pi_e &= \alpha \pi_a + (1 - \alpha) A \\ \alpha(\pi_a) &= \max \left\{ 1 - \frac{1}{B} \left[A(A - \pi_a) - \frac{A^2}{2} + \frac{\pi_a^2}{2} \right]; 0 \right\} \\ E[k | \pi_a \neq \pi] &= \frac{A(A - \pi_a)}{2} + \frac{\pi_a^2}{4} - \frac{A^2}{4} \\ \alpha(\pi_a^*) &> 0\end{aligned}$$

It follows that:

$$\pi_a^* = \arg \max_{\pi_a} \frac{1}{B} \left[B + A\pi_a - \frac{\pi_a^2}{2} - \frac{A^2}{2} \right] \left(\frac{3A^2}{4} - \frac{A\pi_a}{2} - \frac{\pi_a^2}{4} \right) - \frac{3A^2}{4} - \frac{\pi_a^2}{4} + \frac{A\pi_a}{2}$$

The equilibrium, π_a^* , must solve:

$$\begin{aligned}u(\pi_a^*) &= v(\pi_a^*) \\ u(\pi_a) &\equiv \frac{B}{2} (A - \pi_a) \\ v(\pi_a) &\equiv \left(\frac{A + \pi_a}{2} \right) \left(B + A\pi_a - \frac{\pi_a^2}{2} - \frac{A^2}{2} \right) - \left(\frac{3A^2}{4} - \frac{A\pi_a}{2} - \frac{\pi_a^2}{4} \right) (A - \pi_a)\end{aligned}$$

Since $\left[\frac{AB}{2} - A^3 = v(0) < u(0) = \frac{AB}{2} \right]$, $[AB = v(A) > u(A) = 0]$ and $[u'(\cdot) < 0; \text{ and } v'(\cdot) > 0 \forall (\pi_a \leq A)]$, then the equilibrium $\pi_a^* \in (0, A)$ exists and is unique for any $(A > 0, B > 0)$. Now, defining $D(\pi_a^*) \equiv \pi_a^* - A(1 - \alpha(\pi_a^*))$, it is easy to check that $\text{sign}(D(\pi_a^*)) = \text{sign}(2B\pi_a^* + 2A^2\pi_a^* - A^3 - A(\pi_a^*)^2)$. $D(\cdot)$ is positive for $\pi_a = A$ and negative for $\pi_a = 0$. Next we will show that $\text{sign}(D(\pi_a^*))$ is always positive, since π_a^* is lower bounded by some positive value and $D'(\cdot)$ is positive for $\pi_a \in [0, A]$. From $\text{sign}(u(\pi_a) - v(\pi_a)) = \text{sign}((\pi_a)^3 + 2A^3 - \pi_a(3A^2 + 2B))$ we conclude that π_a^* solves $(\pi_a^*)^3 + 2A^3 - \pi_a^*(3A^2 + 2B) = 0$. Then, we can set $\pi_a^* = \frac{2A^3 + (\pi_a^*)^3}{(3A^2 + 2B)}$. Since $D(\pi_a^*)$ is positive whenever $\pi_a^* \geq \frac{A^3}{2B + A^2}$, we must check if $\left(\frac{2A^3 + (\pi_a^*)^3}{(3A^2 + 2B)} \geq \frac{A^3}{2B + A^2} \right)$ holds. It is easy to check that it holds for any $\left(B \geq \frac{A^2}{2} \right)$. Now, if we set $\pi_a = \frac{A^3}{B}$, we have $\text{sign}(u(\pi_a) - v(\pi_a)) = \text{sign}\left(\frac{A^4}{B^2} - 3\right)$. We conclude that $\frac{A^3}{B}$ is a lower bound to π_a^* whenever $\left(B < \frac{A^2}{\sqrt{3}} \right)$. In this case, $\pi_a^* \geq \frac{A^3}{B} \geq \frac{A^3}{2B + A^2}$ and $D(\pi_a^*) > 0$ again. Since $\left(\frac{A^2}{\sqrt{3}} > \frac{A^2}{2} \right)$, we conclude that $\pi_a^* \geq A(1 - \alpha(\pi_a^*))$ for any $(A > 0, B > 0)$. ■

Proof. Proof of proposition 2: The target is fulfilled whenever $w_a = k + h(\alpha) - x \geq 0$, with $x(\pi_a) \equiv \left[A(A - \pi_a) + \frac{\pi_a^2}{2} - \frac{A^2}{2} \right]$. The region for which the target (π_a) may induce multiple equi-

libria expectations is given by the interval $[K^d, K^u]$, where:

$$\begin{aligned}
K^u(\pi_a, \underline{\alpha}) &= \inf \{k \in \mathbb{R} \mid (-x + k + h(\underline{\alpha})) \geq 0\} = x(\pi_a) - h(\underline{\alpha}) \\
K^d(\pi_a, \bar{\alpha}) &= \sup \{k \in \mathbb{R} \mid (-x + k + h(\bar{\alpha})) \leq 0\} = x(\pi_a) - h(\bar{\alpha}) \\
\bar{\alpha} &= \min \left\{ \frac{n - K^d}{\epsilon}; 1 \right\} \text{ if } [K^d, K^u] \cap [n - \epsilon, n] \neq \emptyset \\
\underline{\alpha} &= \max \left\{ \frac{n - K^u}{\epsilon}; 0 \right\} \text{ if } [K^d, K^u] \cap [n - \epsilon, n] \neq \emptyset \\
\bar{\alpha} &= \underline{\alpha} = 0 \text{ if } K^d > n \\
\underline{\alpha} &= \bar{\alpha} = 1 \text{ if } K^u < n - \epsilon
\end{aligned}$$

There are five possible cases for the “[K^d, K^u]-position” related to the support $[n - \epsilon, n]$, as follows:

Case	Exist \Leftrightarrow	K^d	K^u	Equilibrium
1	$x \in [n + h(0), n - \epsilon + h(1)]$	$\in [n - \epsilon, n]$	and $K^u = K^d$	$\alpha \in [0, 1]$
2	$x \in [n + h(0), n - \epsilon + h(1)]$	$\in [n - \epsilon, n]$	$> n$	$\alpha \in [0, 1]$
3	$x \in [n + h(0), n - \epsilon + h(1)]$	$< n - \epsilon$	$\in [n - \epsilon, n]$	$\alpha \in [0, 1]$
4	$n - \epsilon + h(1) > x$		$K^u < n - \epsilon$	$\alpha = 1$
5	$n + h(0) < x$		$K^d > n$	$\alpha = 0$

considering $(h(1) - h(0) \geq \epsilon)$. Otherwise, cases 2 and 3 do not exist and for cases 1, 4 and 5 we set $x \in [n - \epsilon + h(1), n + h(0)]$ instead of setting $x \in [n + h(0), n - \epsilon + h(1)]$. ■

Proof. Proof of proposition 3:

Since $\left(\frac{dx}{d\pi_a}\right) = \pi_a - A$, to relax (to increase) target means to reduce x . From $\Psi(k^*, x) \equiv \Phi^{-1}\left(\frac{k^* + \rho - x}{\rho}\right) - \frac{\sigma}{\sigma_p^2} [k^*] = -\frac{\sigma}{\sigma_p^2} s^p + \Phi^{-1}(1 - c) \frac{\sigma}{\sqrt{\tau}}$ we conclude that $\Psi(\cdot, x)$ is increasing in k^* for every s^p if $\frac{\sigma_p}{\sigma} \leq \sqrt{2\pi}$. Reduction in x must be compensated by reduction in k^* in order to keep $\left[-\frac{\sigma}{\sigma_p^2} s^p + \Phi^{-1}(1 - c) \frac{\sigma}{\sqrt{\tau}} = \Psi(k^*, x)\right]$ valid. The region over the \tilde{k} -support where the target is fulfilled increases for all (s^p) and the size of attack decreases with the decreasing in $s^*(s^p)$.

$\Psi(\cdot, x)$ will be decreasing in k^* for some possible equilibrium $\bar{k}^*(s^p)$ whenever $\frac{\sigma_p}{\sigma} > \sqrt{2\pi}$. In this case, reduction in x must be compensated by an increasing in \bar{k}^* in order to keep $\left[-\frac{\sigma}{\sigma_p^2} s^p + \Phi^{-1}(1 - c) \frac{\sigma}{\sqrt{\tau}} = \Psi(\bar{k}^*, x)\right]$ valid. So, an increase in the target may imply an increasing in \bar{k}^* , $s^*(\bar{k}^*)$, and an increase in the size of attack. ■

Livros Grátis

(<http://www.livrosgratis.com.br>)

Milhares de Livros para Download:

[Baixar livros de Administração](#)

[Baixar livros de Agronomia](#)

[Baixar livros de Arquitetura](#)

[Baixar livros de Artes](#)

[Baixar livros de Astronomia](#)

[Baixar livros de Biologia Geral](#)

[Baixar livros de Ciência da Computação](#)

[Baixar livros de Ciência da Informação](#)

[Baixar livros de Ciência Política](#)

[Baixar livros de Ciências da Saúde](#)

[Baixar livros de Comunicação](#)

[Baixar livros do Conselho Nacional de Educação - CNE](#)

[Baixar livros de Defesa civil](#)

[Baixar livros de Direito](#)

[Baixar livros de Direitos humanos](#)

[Baixar livros de Economia](#)

[Baixar livros de Economia Doméstica](#)

[Baixar livros de Educação](#)

[Baixar livros de Educação - Trânsito](#)

[Baixar livros de Educação Física](#)

[Baixar livros de Engenharia Aeroespacial](#)

[Baixar livros de Farmácia](#)

[Baixar livros de Filosofia](#)

[Baixar livros de Física](#)

[Baixar livros de Geociências](#)

[Baixar livros de Geografia](#)

[Baixar livros de História](#)

[Baixar livros de Línguas](#)

[Baixar livros de Literatura](#)
[Baixar livros de Literatura de Cordel](#)
[Baixar livros de Literatura Infantil](#)
[Baixar livros de Matemática](#)
[Baixar livros de Medicina](#)
[Baixar livros de Medicina Veterinária](#)
[Baixar livros de Meio Ambiente](#)
[Baixar livros de Meteorologia](#)
[Baixar Monografias e TCC](#)
[Baixar livros Multidisciplinar](#)
[Baixar livros de Música](#)
[Baixar livros de Psicologia](#)
[Baixar livros de Química](#)
[Baixar livros de Saúde Coletiva](#)
[Baixar livros de Serviço Social](#)
[Baixar livros de Sociologia](#)
[Baixar livros de Teologia](#)
[Baixar livros de Trabalho](#)
[Baixar livros de Turismo](#)