

EMERGING MARKETS YIELD CURVE DYNAMICS

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Abstract

This work extends Diebold, Li and Yue's (2006) about global yield curve and proposes to extend the study by including emerging countries. The perception of emerging market suffers influence of external factors or global factors, is the main argument of this work. We expect to obtain stylized facts that obey similar pattern found by those authors. The results indicate the existence of global level and global slope factors. These factors represent an important fraction in the bond yield determination and show a decreasing trend of the global level factor low influence of global slope factor in these countries when they are compared with developed countries.

Keywords: Kalman Filter, Emerging Markets, Yield Curve, and Bond.

1 Introduction

The term structure of interest rate stands for the relationship between bond yields and their redemption date, or maturity. Macroeconomic variables and other latent factors determine such relationship and therefore have been studied for a considerable time by academics. The term structure analysis provides a method to extract information from the interaction between those variables, and to forecast how changes in the economic environment may affect the shape of the term structure.

The focus of most studies about yield curve has been on the single-country case using idiosyncratic macroeconomic factors to model the yield. For example, the framework used by Ang and Piazzesi (2003) is the affine models.

The assumption that latent factors generates the yield curve has widely driven the literature on term structure, and some instances are Litterman and Scheinkman (1991); Balduzzi, Das, and Sundaram (1996); Bliss (1997a); and Dai and Singleton (2000). Such factors are usually interpreted as level, slope and curvature, according to Andersen and Lund (1997), Diebold and Li (2006), and Diebold, Rudebusch and Aruoba (2006).

Because the interactions in the global bond markets are very complex, the need to study the relationship in a cross-country environment is enormous. Notwithstanding, it is uncommon to focus on cross-country market, and the exception is Diebold, Li and Yue (2006), who determine the existence of common global yield factors. Specifically, they show the dynamics of cross-country bond interactions using a few countries from OCDE. Their paper follows Nelson and Siegel's (1987) framework, which was extended by Diebold and Li (2006), in such a way that the model is a hierarchical dynamic setting for the countries' yield curve, which depend both on idiosyncratic factors and global factors. Despite the fact the model follows a different framework from others that are in the global environment, such as Solnik (1974) and Thomas and Wickens (1993), it shares similar concerns. Then, Diebold, Li and Yue (2006) determine how the factors are interrelated between each other.

Another important concern is how to measure the global factors. Observed macroeconomic global factors are inadequate to explain the yield curve because each country's macroeconomics measurement methodology may be quite different. On the other hand, any attempt to extract those variables using, for example, principal components analysis is potentially inferior than more structured methodologies that take into account latent variables as the Kalman Filter. Hence, the correct measurement of existing, but latent, global factors is crucial quantify, for example, the country vulnerability or the market integration.

The U.S. market has dominated the empirical literature, but the globalization

process has brought a new dimension up in the world’s bond market. Currently the U.S. bonds correspond to less than 50% of all private and governmental bonds issued around the world. At the same time, the sovereign bonds of emerging markets have increased steadily since their debts has been renegotiated. In search for superior returns and with a long run horizon for investments, pension funds and mutual investment funds has absorbed these bonds. However, in spite of such a growing importance, emerging markets bonds studies have been neglected by the literature. Then, this work comes to bridge such gaps. It uses Diebold, Li and Yue’s (2006) framework to extract global factors related to sovereign bonds from emerging market. The possibility that emerging markets react to global factors is great, but it is neither clear nor well established by any other paper to the best of our knowledge. Therefore, our contribution is to determine whether there exist such global factors, how they explain the term structure dynamics of each country, and compare the results with other existing studies.

The paper is organized as follows. Section (2) presents the theoretical model. Section (4) show the main results, and Section (5) concludes.

2 Theoretical Model

2.1 The Nelson-Siegel’s and Diebold-Li’s Models

Milton Friedman’s claim regarding to the need of a parsimonious model to describe the yield curve has inspired Nelson and Siegel (1987), henceforth NS, to first propose a model for describing that curve. Friedman says: “Students of statistical demand functions might find it more productive to examine how the whole term structure of yield can be described more compactly by a few parameters.” Then, NS follows Friedman’s advice and come up with a simple and parsimonious model, but sufficiently flexible to represent the most common shapes associated with the yield curve: monotonically increasing, humped, and S shaped.

Typical yield curve shapes are generated by a class of functions associated with the solutions of differential and difference equations. For instance, let $d_t(m)$ denote the price of m – *periods* discounted bound, i.e, $d_t(m)$ is the present value at time t of \$1 receivable m periods from today. Let $y_t(m)$ denote the continuously compounded zero-coupon nominal yield to maturity, or spot rate. From the yield curve it is possible to obtain the discount curve:

$$d_t(m) = e^{-m \times y_t(m)} \tag{1}$$

The continuously compounded spot rate is the single rate of return applied until the maturity of m years from today:

$$y_t(m) = -\frac{\ln(d(m))}{m}$$

Another important concept is the *forward rate*, f_t , which measures the prevalent rate in each point in the future. The forward rate average defines the yield to maturity as follows:

$$y_t(m) = \frac{1}{m} \times \int_0^m f_t(x) dx, \quad (2)$$

where $f_t(x)$ denotes the forward rate curve as a function of the maturitie m and $t = 1, 2, \dots, T$.

Hence, from the discount curve (1) and (2) it is possible to obtain the instantaneous (nominal) forward rate curve:

$$f_t(m) = -\frac{d'_t(m)}{d_t(m)} = -[y_t(m) + m \times y'_t(m)]. \quad (3)$$

The heuristic motivation to investigate how the shapes are comes from the expectation theory on the term structure of interest rates. If the spot rates are generated as solutions of a differential equation, then the solution of this equation will be also a forward rates. For instance, NS considered a second order differential equation to describe the movements of the yield curve, and hence, with the assumption of real and unequal roots, the solution will be the forward rate:

$$f_t(m) = b_{0,t} + b_{1,t} \times e^{-\lambda_{1,t}m} + b_{2,t} \times e^{-\lambda_{2,t}m}, \quad (4)$$

where

$\lambda_{1,t}$ and $\lambda_{2,t}$ are time factor loadings associated with the equations;
 $b_{0,t}$, $b_{1,t}$, and $b_{2,t}$ are coefficients to be determined based on initial conditions; and
 $t = 1, \dots, T$.

Hence, the equation (4) gives us a family of forward rate curves whose shapes depend on the values of $b_{1,t}$, and $b_{2,t}$, while $b_{0,t}$ is the asymptote.

There are some problems associated with that function. Depending on the parameters λ_1 and λ_2 , there is more than one value for bs that generate similar curves, such that the bs are not unique. Another problem appears when the convergence is not achieved after performing the nonlinear estimation, what suggests that the function is overparameterized.

In order to overcome such difficulties, NS suggested a more parsimonious model. It generates the same range of shapes of previous specification, but differs from equation (4) by having identical roots as equation (5) shows:

$$f_t(m) = b_{0,t} + b_{1,t} \times e^{-\lambda_t m} + b_{2,t} \times (\lambda_t m \times e^{-\lambda_t m}). \quad (5)$$

The model may be viewed as a constant plus a Laguerre function, that is a polynomial times an exponential decay term, which belongs to a mathematical class of approximating functions¹. Then, the solution for the yield as a function of maturity may be found by solving equation (2):

$$y_t(m) = b_{0,t} + (b_{1,t} + b_{2,t}) \times \frac{(1 - e^{-\lambda_t m})}{\lambda_t m} - b_{2,t} \times e^{-\lambda_t m}. \quad (6)$$

for $t = 1, 2, \dots, T$.

The limiting path of $y(m)$, when m increases, is its asymptote $b_{0,t}$; and, when m is small, the limit is $(b_{0,t} + b_{1,t})$. Different shapes can be drawn by varying the parameters λ and bs . If $\lambda_t = 1$, $b_{0,t} = 1$, $(b_{0,t} + b_{1,t}) = 0$, and $b_{2,t} = a$ where $a \in \mathbb{Z}$, then equation (6) becomes:

$$y_t(m) = 1 - \frac{(1 - a) \times (1 - e^{-m})}{m} - a \times e^{-m}$$

Considering a given period t and by varying the parameter a between -6 and 12 , representing the curves from below to above, the figure shows the possible shapes that one can draw:

¹Please, see details in Abramowitz, Milton and Irene (1965, ch. 22), and Whittaker and Watson (1990, p. 352).

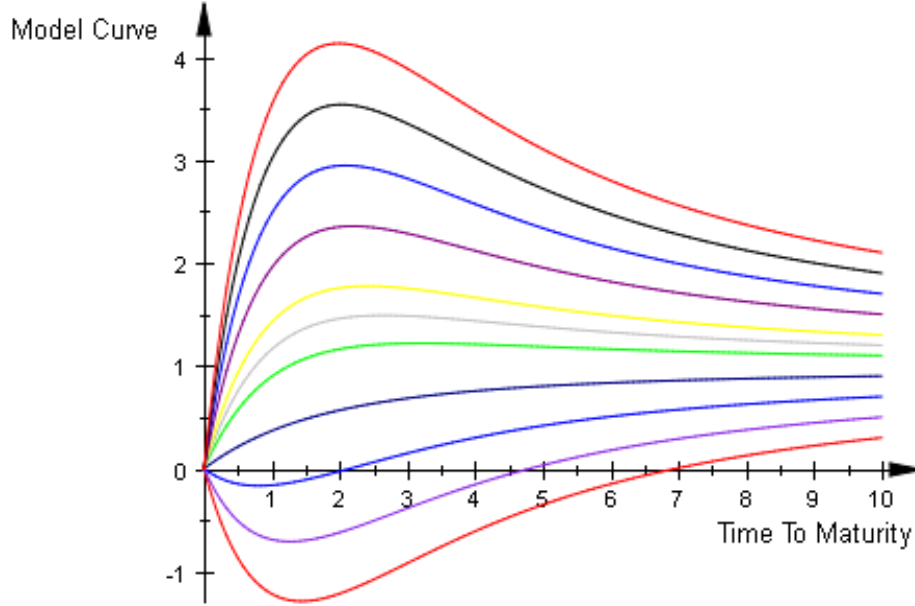


Figure 1: Yield Curve Shapes

NS's model creates two shortcomings. The first is conceptual and claims that it is difficult to give intuitive interpretations for the factors. The second is operational and says it is hard to estimate precisely the factors, because some multicollinearity can emerge. Then, Diebold and Li's (2006), henceforth DL, proposes another factorization:

$$y_t(m) = \beta_{0,t} + \beta_{1,t} \times \frac{(1 - e^{-\lambda_t m})}{\lambda_t m} + \beta_{2,t} \times \left[\frac{(1 - e^{-\lambda_t m})}{\lambda_t m} - e^{-\lambda_t m} \right]. \quad (7)$$

The main distinction is the way they factorize the equation (6). The original NS model matches the DL's when $\beta_{0,t} = b_{0,t}$, $\beta_{1,t} = b_{1,t} + b_{2,t}$, $\beta_{2,t} = b_{2,t}$. DL's factorization is preferable to NS's because both $\frac{(1 - e^{-\lambda_t m})}{\lambda_t m}$ and $e^{-\lambda_t m}$ have similar decreasing shapes, and, if $b_{1,t}$ and $b_{2,t}$ are interpreted as factors, their respective loadings, $\frac{(1 - e^{-\lambda_t m})}{\lambda_t m}$ and $e^{-\lambda_t m}$, would be very similar.

The factor loadings 1, $\frac{(1 - e^{-\lambda_t m})}{\lambda_t m}$ and $e^{-\lambda_t m}$, can be easily extracted using the maturities m and a specific constant λ_t . They can be interpreted as measuring the

strength of long, medium and short term components of the forward rate or of the yield curve.

The parameter λ is related to the exponential decay rate. Small values of λ produce slow decay, and fit well long term maturities; by contrast, large values of λ produce fast decay and fit better curves that have short term maturities. DL choose a constant value for λ , in such a way that to maximize the curvature loading. Since, it is usually observed that maturity finds the maximum point between 2 and 3 years², DL use the average between these two maturities and set $\lambda = 0.0609$, corresponding to 30 months.

The shape of the factor loadings, 1 , $\frac{(1-e^{-\lambda m})}{\lambda m}$, and $\left[\frac{(1-e^{-\lambda m})}{\lambda m} - e^{-\lambda m}\right]$ are illustrated in figure (2). As mentioned before, the factor corresponding to $\beta_{0,t}$ represents the long term, the one corresponding $\beta_{2,t}$ represents the medium term, and the last one corresponding to $\beta_{1,t}$ represents the short term.

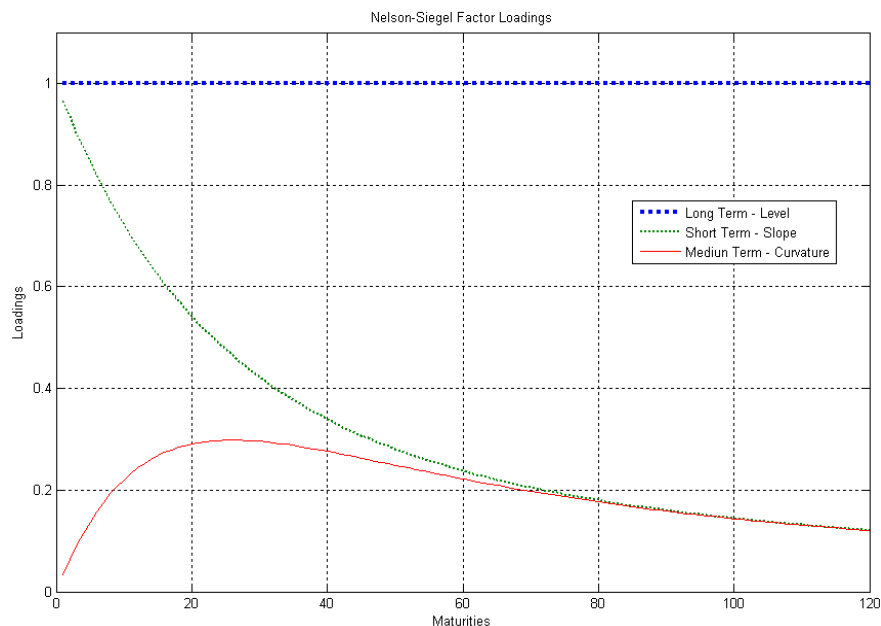


Figure 2: Nelson-Siegel Factor Loadings

²See that in figure 2.

3 The Global Model

Last section showed that studies of the U.S. closed-economy environment using a generalized DL model fit well the dynamics of the yield curve. Now, in this section the model will be extended to a multi-country environment, following Diebold, Li and Yue (2006), henceforth DLY.

Using the DL factorization of the NS yield curve for a single country and indexing the parameters to represent a specific country, the model is:

$$y_{i,t}(m) = l_{i,t} + s_{i,t} \times \frac{(1 - e^{-\lambda_{i,t}m})}{\lambda_{i,t}m} + c_{i,t} \times \left[\frac{(1 - e^{-\lambda_{i,t}m})}{\lambda_{i,t}m} - e^{-\lambda_{i,t}m} \right] + \varepsilon_{i,t}(m), \quad (8)$$

where

$y_{i,t}(m)$ is the continuously-compounded zero-coupon nominal yield of a bond maturing m periods ahead, in country i at period t ;

$i = 1, 2, \dots, N$, and $t = 1, 2, \dots, T$;

$\varepsilon_{i,t}(m)$ represents a disturbance with variance $\sigma_i^2(m)$.

The coefficients are interpreted as latent factors. They are the level, the slope, and the curvature, denoted, respectively, by l , s , and c .

The previous model was simplified by DL as shown in equation (9). The first simplification makes $\lambda_{i,t}$ constant across countries and time. The authors argue there is a tiny loss of generality from doing that, since λ determines the maturity at which the curvature loading reaches the maximum. The second simplification makes $c_{i,t} = 0$ for all t and i . The argument for doing that comes from the fact that missing data makes the estimated curve be considerably imprecise at very short and/or very long maturities. They also allege that the curvature is not associated with macroeconomic fundamentals, as level is connected to inflation and slope is connected with GDP or capacity of utilization. Hence, the model can be written as:

$$y_{i,t}(m) = l_{i,t} + s_{i,t} \times \frac{(1 - e^{-\lambda m})}{\lambda m} + \varepsilon_{i,t}(m). \quad (9)$$

In Diebold, Rudebusch and Aruoba (2006), the single-country version was expressed in terms of a space-state framework, such that equation (9) represents the space equation, and the time varying parameters l_{it} and s_{it} , which follow a first-order diagonal autoregression vector, represents the state equations.

From the single-country model, one may adapt it to an N -country approach, coupled with a similar space-state framework. The problem now is that $Y_t(m)$ is not observed as well as the factors, that is:

$$Y_t(m) = L_t + S_t \times \frac{(1 - e^{-\lambda m})}{\lambda m} + v_t(m) \quad (10)$$

where

$Y_t(m)$ is a theoretical global yield;
 L_t is the global level; and
 S_t is the global slope.

These latent global factors are common to every country. It is postulated that the global yield factors follow a first-order VAR model as follows:

$$\begin{pmatrix} L_t \\ S_t \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} L_{t-1} \\ S_{t-1} \end{pmatrix} + \begin{pmatrix} U_t^l \\ U_t^s \end{pmatrix} \quad (11)$$

where

U_t^n is the structural disturbance $n = l, s$; and

$$E [U_t^n U_{t'}^{n'}] = \begin{cases} (\sigma^n)^2, & \text{if } t = t' \text{ and } n = n' \\ 0, & \text{otherwise} \end{cases}.$$

Then the model decomposes the country-specific level (slope) into a global level (slope) and some idiosyncratic factor, $\varepsilon_{i,t}^n$:

$$l_{i,t} = \alpha_i^l + \beta_i^l L_t + \varepsilon_{i,t}^l, \quad (12a)$$

$$s_{i,t} = \alpha_i^s + \beta_i^s S_t + \varepsilon_{i,t}^s, \quad (12b)$$

for every $i = 1, 2, \dots, N$.

Equation (11) assumed that global common factor followed a first-order VAR. Here it will be allowed that the country idiosyncratic factors share the same characteristic.

$$\begin{pmatrix} \varepsilon_{i,t}^l \\ \varepsilon_{i,t}^s \end{pmatrix} = \begin{pmatrix} \theta_{i,11} & \theta_{i,12} \\ \theta_{i,21} & \theta_{i,22} \end{pmatrix} \begin{pmatrix} \varepsilon_{i,t-1}^l \\ \varepsilon_{i,t-1}^s \end{pmatrix} + \begin{pmatrix} u_{it}^l \\ u_{it}^s \end{pmatrix} \quad (13)$$

Where

u_{it}^n is the disturbance $n = l, s$; and

$$E [u_{it}^n u_{it'}^{n'}] = \begin{cases} (\sigma_2^n)^2, & \text{if } i = i' \text{ and } t = t' \\ 0, & \text{otherwise} \end{cases}$$

In terms of a state-space model, Equations (11) and (13) are transition equations.

The multi-country yield curve model was represented in state-space framework, now, joint the measurement equation for single country in matrix notation, obtains:

$$\begin{bmatrix} y_{1t}(m_1) \\ y_{1t}(m_2) \\ \vdots \\ y_{Nt}(m_{J-1}) \\ y_{Nt}(m_J) \end{bmatrix}_{JN \times 1} = A \times \begin{bmatrix} \alpha_1^l \\ \alpha_1^s \\ \vdots \\ \alpha_N^l \\ \alpha_N^s \end{bmatrix}_{2J \times 1} + C \quad (14a)$$

$$C = B \times \begin{bmatrix} L_t \\ S_t \end{bmatrix}_{2 \times 1} + A \times \begin{bmatrix} \epsilon_{1t}^l \\ \epsilon_{1t}^s \\ \vdots \\ \vdots \\ \epsilon_{Nt}^l \\ \epsilon_{Nt}^s \end{bmatrix}_{2J \times 1} \quad (14b)$$

Where, N is the numbers of countries, J is the number of maturities and, A , B and C are:

$$A = \begin{pmatrix} 1 & \frac{1-e^{-m_1\lambda}}{m_1\lambda} & 0 & \dots & 0 & 0 \\ 1 & \frac{1-e^{-m_2\lambda}}{m_2\lambda} & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & 1 & \frac{1-e^{-m_{J-1}\lambda}}{m_{J-1}\lambda} \\ 0 & 0 & \dots & \dots & 1 & \frac{1-e^{-m_J\lambda}}{m_J\lambda} \end{pmatrix}_{JN \times 2J} \quad (14c)$$

$$B = \begin{pmatrix} \beta_1^l & \beta_1^s \left(\frac{1-e^{-m_1\lambda}}{m_1\lambda} \right) \\ \beta_1^l & \beta_1^s \left(\frac{1-e^{-m_2\lambda}}{m_2\lambda} \right) \\ \dots & \dots \\ \dots & \dots \\ \beta_N^l & \beta_N^s \left(\frac{1-e^{-m_{J-1}\lambda}}{m_{J-1}\lambda} \right) \\ \beta_N^l & \beta_N^s \left(\frac{1-e^{-m_J\lambda}}{m_J\lambda} \right) \end{pmatrix}_{JN \times 2} \quad (14d)$$

$$C = \begin{bmatrix} \epsilon_{1t}(m_1) \\ \epsilon_{1t}(m_2) \\ \vdots \\ \vdots \\ \epsilon_{Nt}(m_{J-1}) \\ \epsilon_{Nt}(m_J) \end{bmatrix}_{JN \times 1} \quad (14e)$$

It is important to note that the global common factor, β' s and the factor loading, $\left(\frac{1-e^{-m_{J-1}\lambda}}{m_{J-1}\lambda}\right)$ are not separately identified³. Because of this it will be considered that β_{BR}^n is positive, that is $\beta_{BR}^n > 0$ and $n = l, s$; and because of the magnitudes of global factors and factor loadings the innovations to global factors have unit standard deviation, that is, $\sigma^n = 1, n = l, s$ ⁴.

³In equation (14d)

⁴This follow Sargent and Sims (1977) and Stock and Watson (1989).

3.1 Econometric Estrategy

The estimation method in a multi-country enviroment can be done using the equation(14a,and 14b). The state-space can be estimated by Kalman Filter, and fully-efficient Gaussian maximum likelihood dynamics estimates are obtained. In the single-country situation, to estimate the latent factors using Kalman Filter framework is relatively easy, because the number of parameters is small. In a multi-country situation, however, one-step maximum likelihood is difficult to implement, due to the large number of parameters for estimate. Hence, DLY propose a convenient multi-step estimation method.

The first step is to obtain the latent factors (level and slope) for each country. The second step is to use estimates previously obtained and use equations(11,12a,12b, and 13) to extract the global factors.

3.1.1 Single Country Estimation

In equation (6) are able to see that λ is time varying, following the original NS model. So, the parameters ($l_{i,t}$, $s_{i,t}$ and λ_t) are able to estimate using nonlinear least square for each month t . However DL use different strategy. They fix the parameter λ at a specific value. In the first moment they compute two regressors (factor loadings⁵) for many maturities, and they use ordinary least square in these cross-section data to estimate the parameters ($l_{i,t}$ and $s_{i,t}$), for each month. Hence, there are two parameters for each month, which are $l_{i,t}$ and $s_{i,t}$ for specific country i and specific date t .

Furthermore, DL's model can be represented as a space state system, where the latent factors represent the state and the main formula the space. This method is used to estimate the parameters (l_{it} and s_{it}) can be obtained using the state-space system, following Diebold-Rudebush-Aruoba(2006). In general, the space-state system is a powerful framework for estimation of dynamic models, because it is possible to apply the Kalman Filter that provides maximum-likelihood estimates of factors. In the equation (9), $l_{i,t}$ and $s_{i,t}$ follow a vector autoregressive⁶ process of first order, and because of this, the model became a state-space system. The transition equation or the state equation, which governs the dynamics of state vector⁷, is:

⁵The factor loadings are not stochastic and they are obtained using $\frac{1-e^{-m_N\lambda}}{m_N\lambda}$ for slope factor and level is 1 for all maturities, as the figure(2).

⁶It will be maintain the $VAR(1)$ assumption for transparency and parsimony.

⁷The initial values of coefficients to proceed this estimation will be obtained in Diebold-Li (2006) method using unconditional $VAR(1)$ in level and slope factors. The $VAR(1)$ diagonal principal (a_{11} and a_{22}) of coefficients matrix are the initial coefficients.

$$\begin{pmatrix} l_{i,t} - \mu_{i,l} \\ s_{i,t} - \mu_{i,s} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} l_{i,t-1} - \mu_{i,l} \\ s_{i,t-1} - \mu_{i,s} \end{pmatrix} + \begin{pmatrix} n_{i,t}(l) \\ n_{i,t}(s) \end{pmatrix} \quad (15)$$

Where

$i = 1, \dots, N$ is the specific country, and $t = 1, \dots, T$.

The set of yield curves equations with a set of N yields and the three unobservable factors, is:

$$\begin{pmatrix} y_{i,t}(m_1) \\ y_{i,t}(m_2) \\ \cdot \\ \cdot \\ y_{i,t}(m_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-m_1\lambda}}{m_1\lambda} \\ 1 & \frac{1-e^{-m_2\lambda}}{m_2\lambda} \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & \frac{1-e^{-m_N\lambda}}{m_N\lambda} \end{pmatrix} \begin{pmatrix} l_{i,t} \\ s_{i,t} \end{pmatrix} + \begin{pmatrix} \epsilon_{i,t}(m_1) \\ \epsilon_{i,t}(m_2) \\ \cdot \\ \cdot \\ \epsilon_{i,t}(m_N) \end{pmatrix} \quad (16)$$

In the vector/matrix notation, the state-space system is:

$$(x_{i,t} - \mu_i) = A(x_{i,t-1} - \mu_i) + \eta_{i,t} \quad (17a)$$

$$y_{i,t} = \Lambda x_{i,t} + \epsilon_{i,t} \quad (17b)$$

The Kalman Filter is used to provide the least square estimators of state vectors, and it requires that the white noise transition and measurement disturbance be orthogonal to each other and to initial states.

$$\begin{pmatrix} \eta_{i,t} \\ \epsilon_{i,t} \end{pmatrix} \sim WN \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right] \quad (18)$$

$$E(f_0 \eta'_{i,t}) = 0 \quad (19)$$

$$E(f_0 \epsilon'_{i,t}) = 0 \quad (20)$$

The analysis assumes that the H matrix is diagonal and this assumption implies that the deviations of yields of various maturities from yield curve are uncorrelated, that is quite standard. The Q matrix is not assumed to be diagonal or unrestricted, and this allows that the shocks of the two latent factors be correlated.

The Kalman filter single country factor extraction provides similar results than DL method, and the differences are not more than 1%, because of this, for convenience the level and slope will be extracted using DL.

3.1.2 Multi-Country Estimation

The second step or multi-country version model is estimated by Kalman Filter. At this moment a big difficulty arises. The initial values must be obtained. To obtain them, first of all it will be necessary to extract the principal components of level ($l_{i,t}$) and slope ($s_{i,t}$) series, obtained in the first step for all countries, as a proxy of global level factor (L_t^{PCA}) and global slope factor (S_t^{PCA}). In the model there are two initial values for global factor and two initial values idiosyncratic factor for each country. The total initial values are $2 + 2N$ initial values, where N is a number of countries.

Hence, it will be proceed the unconditional $VAR(1)$:

$$\begin{pmatrix} L_t^{PCA} \\ S_t^{PCA} \end{pmatrix} = \begin{pmatrix} \phi_{11}^{PCA} & \phi_{12}^{PCA} \\ \phi_{21}^{PCA} & \phi_{22}^{PCA} \end{pmatrix} \begin{pmatrix} L_{t-1}^{PCA} \\ S_{t-1}^{PCA} \end{pmatrix} + \begin{pmatrix} Z_t^l \\ Z_t^s \end{pmatrix}$$

There are not evidences that level is correlated with slope, or vice-versa, because of this it will be used only the diagonal principal of coefficients, the parameters, ϕ_{11}^{PCA} and ϕ_{22}^{PCA} .

The initial values for idiosyncratic factors are obtained for each country using the principal components again. First of all, the following equations must be regressed:

$$\begin{aligned} l_{i,t} &= c_i^l + \delta_i^l \times L_t^{PCA} + \varepsilon_{i,t}^{PCA,l} \\ s_{i,t} &= c_i^s + \delta_i^s \times S_t^{PCA} + \varepsilon_{i,t}^{PCA,s} \end{aligned} \quad (21)$$

Where $l_{i,t}$ and $s_{i,t}$ are vectors of the level and slope for all countries obtained in the first step.

The errors $\varepsilon_{i,t}^{PCA,l}$ and $\varepsilon_{i,t}^{PCA,s}$ are collected, and the new $VAR(1)$ must be proceed.

$$\begin{pmatrix} \varepsilon_{i,t}^{PCA,l} \\ \varepsilon_{i,t}^{PCA,s} \end{pmatrix} = \begin{pmatrix} \theta_{i,11}^{PCA} & \theta_{i,12}^{PCA} \\ \theta_{i,21}^{PCA} & \theta_{i,22}^{PCA} \end{pmatrix} \begin{pmatrix} \varepsilon_{i,t-1}^{PCA,l} \\ \varepsilon_{i,t-1}^{PCA,s} \end{pmatrix} + \begin{pmatrix} r_{i,t}^l \\ r_{i,t}^s \end{pmatrix}$$

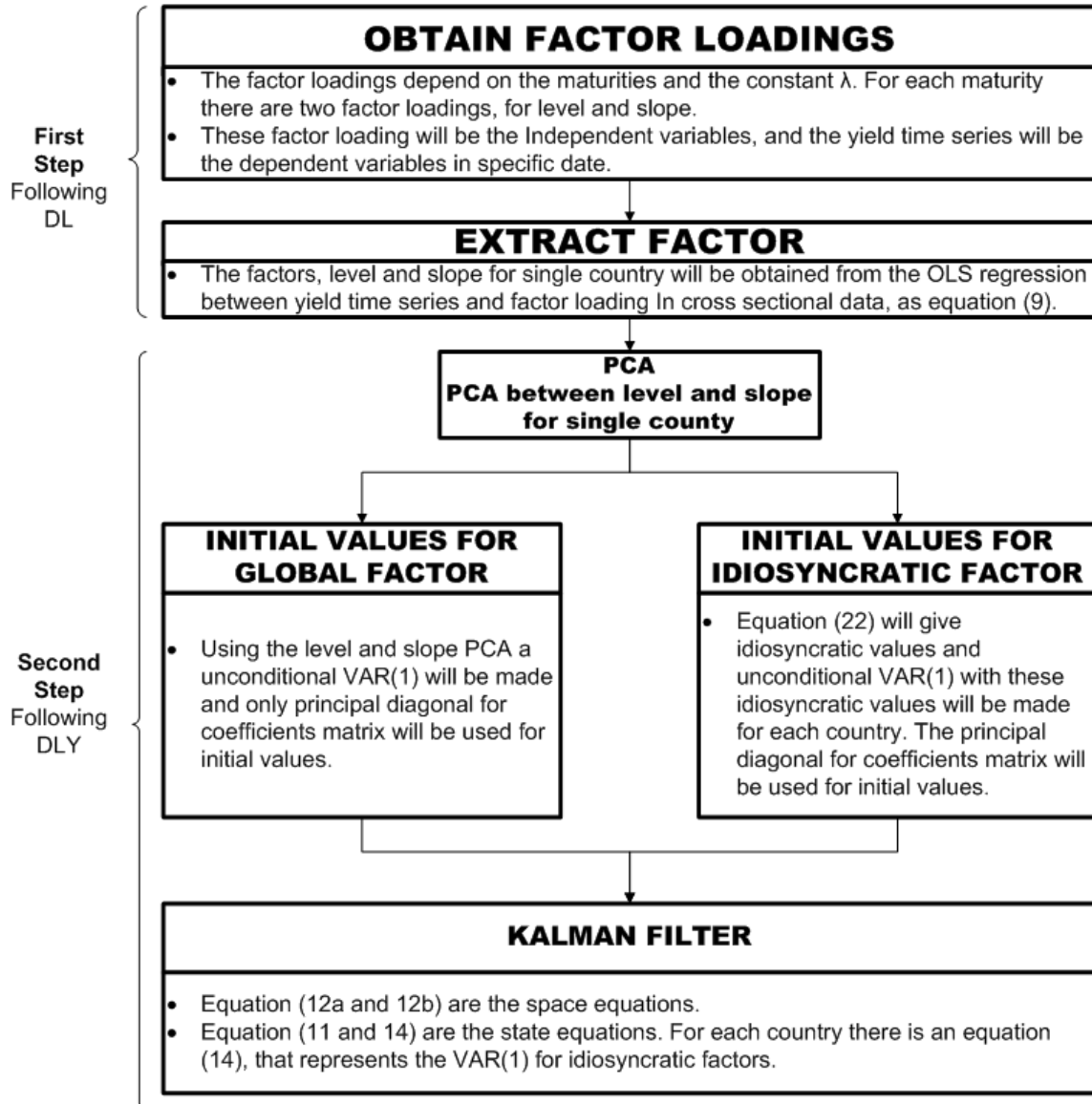
Again, there are not evidences that idiosyncratic level is correlated with idiosyncratic slope, or vice-versa, because of this it will be used only the diagonal principal of coefficients, the parameters, $\theta_{i,11}^{PCA}$ and $\theta_{i,22}^{PCA}$.

Hence, set of equations (??), (11) and (13) that follow the space-state system will be estimated using Kalman Filter, and the set of decomposition equation (12a, and 12b, or (??)) are the measurement equation or space equations, and the autoregressive vectors equations (11) and (13) are the transition equations or state equations.

For each factor there are one autoregressive coefficient for the global factor ϕ_{kj} , N intercepts (α), N loadings for global factor (β), N autoregressive coefficients for

idiosyncratic factors $\theta_{i,kj}$, and N standard deviation for innovations to the idiosyncratic factors. Where N is the number of country. Hence the number of coefficients that will be estimated are $2 + 8N$.

ECONOMETRIC ESTRATEGY – MULTI-STEP ESTIMATION



4 EMPIRICAL RESULTS

4.1 Data Analysis

4.1.1 Data Construction

This work's data consists of government zero-coupon bond yields. The bonds are in terms of US currency and the country's bonds that will be analyzed are Brazil, South Korea, and Mexico. The choice of these countries deserves one more explanation. Mexico is highly connected with US and can be used as a US proxy, because US tend to affect all countries in the world. Korea is a control variable because of the distance of choice set. And Brazil is a point of analysis. Another point to analyze these countries comes from the fact they are countries that recently fight against economics crises, and today are considered economies in development. Such characteristic suggests that they have large global influence when they issue or when negotiate their bonds. The data used here begin in June 7th of 1998 and extend to September 9th of 2007, and they are monthly data that are measured in the last day of month. Because of the difference in the maturities in the countries it is important to introduce the concept of interpolation⁸ to homogenizes the data.

The yield is observed with discrete maturities, and they can be different in the set of bonds. In the fixed income analysis is it common to extract the rates for the maturity that is not observed and it can be made using the interpolation method. It is known that the discount rate is a monotonically decreasing function of maturity and the price of bonds can be expressed as a linear combination of discount rates. McCulloch(1971,1975) suggest that a spline method could be used to interpolate the discount function or the bond price directly.

4.1.2 Data Description

In this work it will be consider monthly zero-coupon bond yields. The maturities that will be used is fixed in 3, 6, 12, 24, 36,48, and 72 months, and they are obtained in the cubic spline process.

⁸More information in Appendix

Brazil								
Maturity	Mean	Std. Deviation	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	Max.	Min.	
3	6.98	4.95	0.769	0.037	-0.068	33.49	0.99	
6	7.19	4.66	0.821	0.069	-0.075	33.11	1.09	
12	7.57	4.68	0.823	0.089	-0.079	33.34	2.02	
24	8.59	4.52	0.804	0.131	-0.104	33.85	4.06	
36	9.85	4.79	0.887	0.180	-0.036	33.12	4.98	
48	10.83	5.01	0.897	0.181	-0.043	31.71	5.22	
60	11.38	5.05	0.897	0.181	-0.043	31.63	5.35	
72	11.41	4.90	0.894	0.191	-0.052	30.92	5.40	
South Korea								
Maturity	Mean	Std. Deviation	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	Max.	Min.	
3	4.84	2.57	0.958	0.514	-0.045	13.20	1.16	
6	5.01	2.63	0.957	0.502	-0.045	13.82	1.29	
12	5.25	2.56	0.957	0.510	-0.050	13.93	1.59	
24	5.58	2.44	0.953	0.512	-0.056	14.06	2.38	
48	6.22	2.26	0.946	0.510	-0.033	14.45	3.40	
60	6.34	2.21	0.942	0.490	-0.028	14.65	3.22	
72	6.43	2.18	0.942	0.482	-0.027	14.79	3.70	
Mexico								
Maturity	Mean	Std. Deviation	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	Max.	Min.	
3	4.28	2.19	0.913	0.487	-0.070	13.30	1.07	
6	4.47	2.22	0.910	0.498	-0.083	13.95	1.12	
12	4.75	2.19	0.914	0.533	-0.097	14.17	1.25	
24	5.29	2.12	0.910	0.567	-0.108	14.39	1.50	
36	5.89	2.02	0.903	0.562	-0.072	14.48	2.46	
48	6.39	1.98	0.907	0.609	-0.040	14.41	3.40	
60	6.83	1.97	0.917	0.602	-0.022	14.19	4.13	
72	7.093	1.90	0.930	0.604	-0.001	13.31	4.63	

Maturity in months

Table 1: Summary Statistics for Yield Bond

It is possible to note that Brazilian's interest rates are highest that others on average, approximately 6.5%. The Brazil, South Korea and Mexico's yield curves are on the average upward-sloping. The volatility tends to decrease with maturities for all countries, with exception in Brazil's 48-years maturity. All yields are highly persistent for all countries, in Brazil and Mexico the persistence tend to increase with

the maturities, in the opposite of South Korea, and the average of first autocorrelation is 0.87.

The figure (3,4 and 5) show the government bond yield curves over countries and time. It is difficult to evaluate the bond yield curves in cross-country analysis, apparently the difference are in the level of yield. The higher level certainly is in Brazil's bond, which revels great increase near 2002. This crise was named Lula's effect. Lula was a principal candidate in the election, and his ideology was opposite of that was considered a good economy policy. It is important to note that South Korea and Mexico yield curves are better-behave in comparison with Brazil's yield curve.

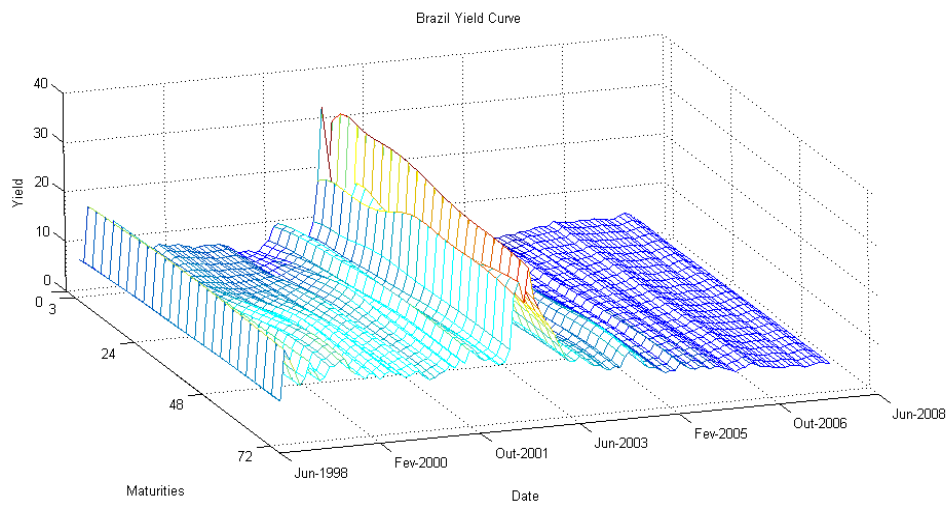


Figure 3: Yield Curve over Space and Time

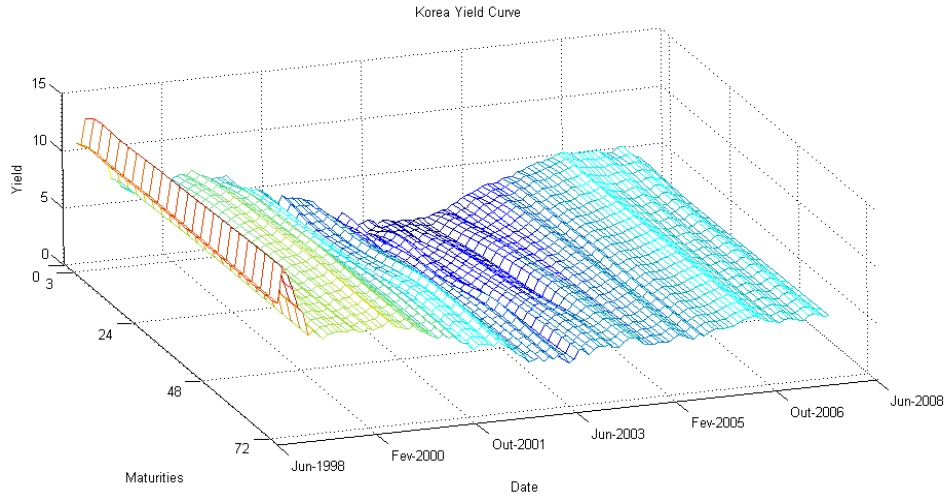


Figure 4: Yield Curve over Space and Time

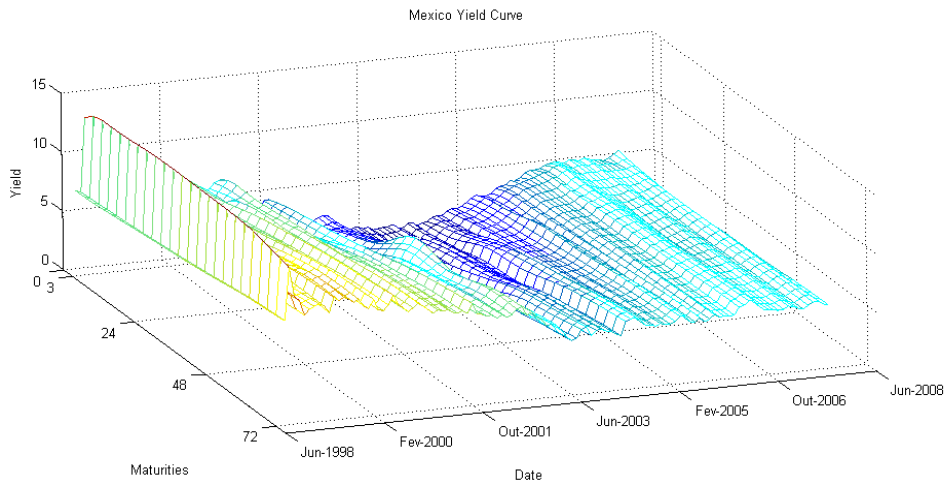


Figure 5: Yield Curve over Space and Time

4.1.3 Preliminary Analysis

The multi-step estimation generated a lot of series and estimations that can be analyzed. Many of these estimates have important intuition about the desirable final results. In the first step of the model, it was extract the estimates level and slope, $\{l_{it}$ and $s_{it}\}$, $t = 1, \dots, T$ and $i = 1, \dots, N$, following DL's two step method. The figure of these series can be viewed in (6) and (7).

The descriptive statistic for estimated factor indicates an autocorrelation the persistent dynamics. In general, the level factor correlograma indicates that its revel more persistent that slope factor. The Augmented Dickey-Fuller test did not reject the existence of unit root in both of factors in all countries. The existence of unit root is controversy in the nominal bond yields. The theory shows that the root is less that one, because, if it does not occurs, the nominal bond yield would eventually go to negative, that is not an acceptable. In DBY's work, similar results are obtained.

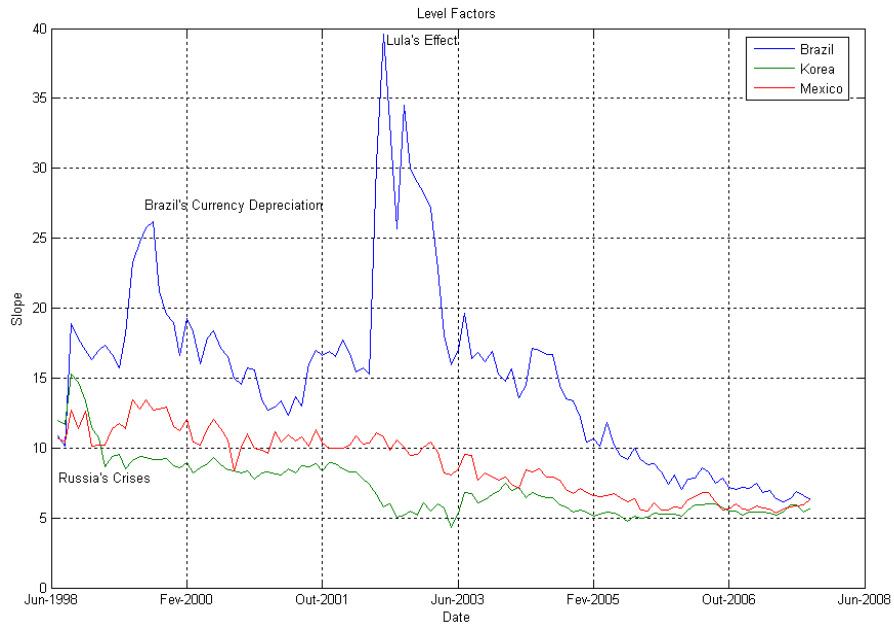
Brazil								
Factor	Mean	Std. Dev.	Max.	Min.	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	<i>ADF</i>
$\hat{l}_{i,t}$	13.13	5.57	31.43	5.41	0.9	0.228	-0.03	-1.88
$\hat{s}_{i,t}$	-7.64	4.90	2.26	-20.39	0.79	0.14	0.014	-1.77
South Korea								
Factor	Mean	Std. Dev.	Max.	Min.	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	<i>ADF</i>
$\hat{l}_{i,t}$	6.86	2.17	15.00	4.12	0.93	0.47	-0.01	-2.39
$\hat{s}_{i,t}$	-2.37	1.70	0.58	-5.70	0.94	0.26	0.09	-1.28
Mexico								
Factor	Mean	Std. Dev.	Max.	Min.	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	<i>ADF</i>
$\hat{l}_{i,t}$	7.65	2.14	14.29	5.04	0.93	0.54	0.031	-1.89
$\hat{s}_{i,t}$	-4.22	2.58	0.25	-8.77	0.91	0.21	0.09	-1.56

ADF Denotes an argumented Dicky-Fuller test Statistic

Table 2: Descriptive Statistics for Estimated Factors

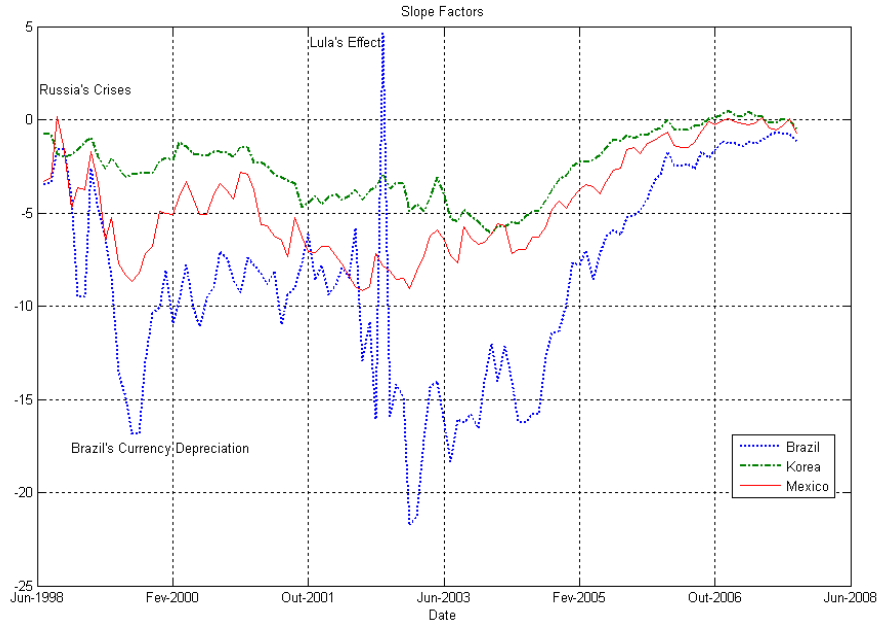
The graphics of the factors are plotted separately by factor. The graphic of both factors indicate that Brazil suffer a big oscillation in 2002, when other did not have great variation in the period⁹.

⁹These graphics can be viewed in sub-periods in the appendix B.



The estimation follow DL described in the text

Figure 6: Level Factors using DL



The estimation follow DL described in the text

Figure 7: Slope Factor using DL

An important insight arises when these figures are analyzed. In the internal crises periods is important to note that both, level and slope factors are not connected across countries. Lula's crisis in 2002 is the best example of this fact. During international crises, like Russia's crises in 1998 it is possible to note that both, level and slope are connected. This characteristic infer that in the normal period, like after 2006 the curves are linked, and it explain the actual expectation in Brazil to obtain the investment grade from risk agencies, investment grade already obtained for Mexico and korea. The curves show also that the level, that is linked with inflation, of yields are converging for stability and the same convergency is obtained for slope factor, that is linked with GDP.

The analysis of these curves can infer about the global factor results. The global factor are extracted take in account that existence of connections between the curves. DLY used the set of developed countries and the comparative analysis between these two set of countries it is possible to note that the oscilations in level and slope factor in developed markets is smoothest; the opposite is expected to occur in emerging markets, because of the internal oscillation in the curve.

The importance of putting the graphics together is to note that similarity of the

movements. Despite of the variability, it is possible to note that the dynamics of the factors run in the same directions. This evidence can be better described using the principal components framework. The PCA (principal components analysis) indicate the existence of global level and slope factor. The first principal component for level explains more than eighty percent of variation and the first principal component of slope explain more that ninety percent of variation.

	Comp 1	Comp 2	Comp 3
Eigenvalue	32.62	6.97	0.36
Variance Prop.	0.82	0.17	0.01
Cumulative Prop.	0.82	0.99	1

Table 3: Principal Components Analysis for Estimated Level Factor

	Comp 1	Comp 2	Comp 3
Eigenvalue	30.31	2.34	0.48
Variance Prop.	0.92	0.07	0.01
Cumulative Prop.	0.92	0.99	1

Table 4: Principal Components Analysis for Estimated Slope Factor

4.2 Results

The level and slope factors used are provided in the first step regression, using DL's method. In the second step it was used the Kalman Filter to evaluate the likelihood function which the maximization is made by Marquart algorithm with convergence criteria of 0.0001.

The Kalman filter was initialized using the diagonal covariance matrix of state vector, and the initial parameters was choose using the $VAR(1)$ processes of first step using the level and slope principal components series in place of the latent global yield factors.

The estimation results indicate that the global yield factor are highly serially correlated, and all country level yield factors load positively and significantly on the correspondent global yield factors. In the slope, the impact is negative and significant. The country idiosyncratic factor, like a global factor, is serially correlated, but less than the global factors. Although, the global and idiosyncratic are significant, the constant terms are only in level factor for South Korea and Mexico.

Global Level Factor		Global Slope Factor	
$L_t = 0.95 \times L_{t-1} + U_t^l$ (0.05)*		$S_t = 0.95 \times S_{t-1} + U_t^s$ (0.06)*	
Country Level Factor			
$l_{B,t} = 11.99 + 1.32 \times L_t + \varepsilon_{B,t}^l$ (9.60) (0.34)*	$\varepsilon_{B,t}^l = 0.94 \times \varepsilon_{B,t-1}^l + 1.00 \times v_{B,t}^l$ (0.05)* (0.12)*		
$l_{K,t} = 7.84 + 0.40 \times L_t + \varepsilon_{K,t}^l$ (2.55)* (0.07)*	$\varepsilon_{K,t}^l = 0.98 \times \varepsilon_{K,t-1}^l - 2.03 \times v_{K,t}^l$ (0.03)* (0.15)*		
$l_{M,t} = 7.54 + 0.65 \times L_t + \varepsilon_{M,t}^l$ (3.08)** (0.06)*	$\varepsilon_{M,t}^l = -0.57 \times \varepsilon_{M,t-1}^l - 3.68 \times v_{M,t}^l$ (0.25)** (0.80)*		
Country Slope Factor			
$s_{B,t} = -6.30 - 1.51 \times S_t + \varepsilon_{B,t}^s$ (7.43) (0.23)*	$\varepsilon_{B,t}^s = -0.38 \times \varepsilon_{B,t-1}^s + 0.76 \times v_{B,t}^s$ (0.12)* (0.24)*		
$s_{K,t} = -1.96 - 0.24 \times S_t + \varepsilon_{K,t}^s$ (1.44) (0.07)*	$\varepsilon_{K,t}^s = 0.94 \times \varepsilon_{K,t-1}^s - 2.06 \times v_{K,t}^s$ (0.05)* (0.28)*		
$s_{M,t} = -3.64 - 0.60 \times S_t + \varepsilon_{M,t}^s$ (2.97) (0.13)*	$\varepsilon_{M,t}^s = 0.90 \times \varepsilon_{M,t-1}^s - 0.90 \times v_{M,t}^s$ (0.05)* (0.21)*		

Note-The numbers in the parenthesis represents the standard Deviation , One asterisk Indicate that is significant at 1 percent, two asterisk Indicate that is significant at 5 percent

Table 5: Estimates of Global Yield Curve Models

In figure(8) below it is possible to note that the Global level is high correlated with level's country principal components. The correlation with both series is near fifty seven percent. In the next figure(9) the Global Slope not is very correlated with slope's principal components. The correlation is negative and is near of twenty eight percent.

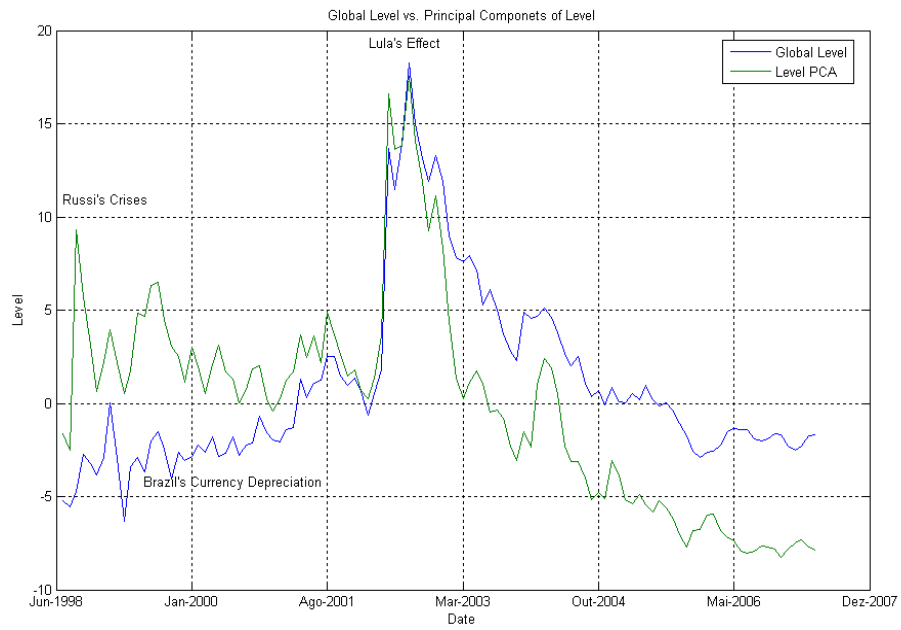


Figure 8: Global Level Factor vs. PCA of Level

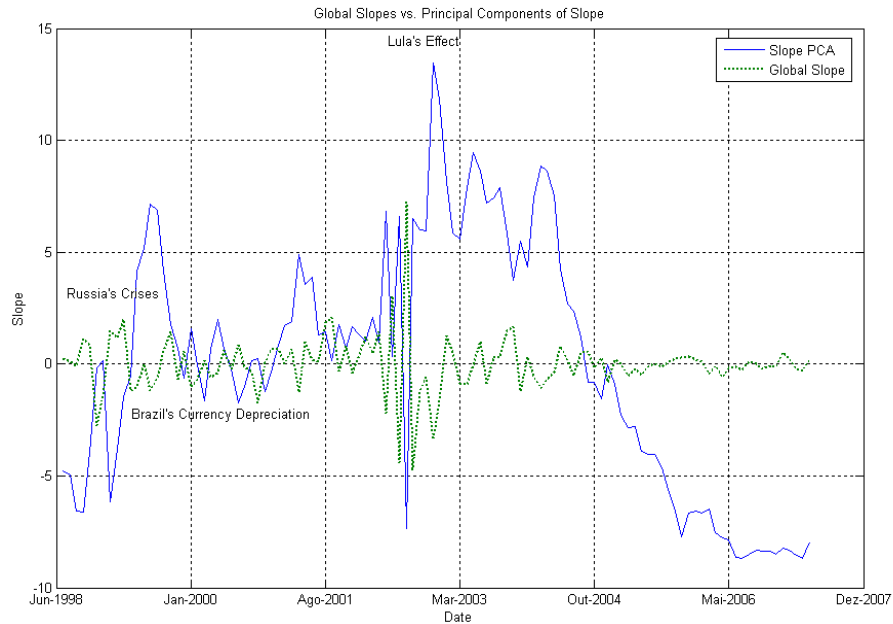


Figure 9: Global Slope Factor vs. PCA of Slope

Some interesting conclusions can be made analyzing table of result and graphics. The level factor influence Brazil's level more than two other country, the influence in the level factor is 1.32 while in Korea and Mexico are 0.40 and 0.65 respectively. For instance, in some date of global level, the index can be near from 9.00, so in Brazil the influence is 32% higher than global level, and in Korea and Mexico 40% and 65% less than level yield curve. Another conclusion about the graphic of global level curve is that there is a tendency to negative numbers that mean that the level of yield decreased during the period. Some similar characteristic can be found in DLY for global level curve. The slope curve in all countries suffers low influence for global slope factor because the curve is around the zero and the result is very different of DLY. The answer key to this question comes from the level of yield in these countries. The slope factor can be described as some sort run tendency of yield curve. In developed countries the level yield are lower than emerging countries and because of this the tendency for changes in the level in sort run or even the volatility tend to be higher than in emerging countries.

The discussion about the link between macroeconomic variables and latent factors, in special, level and slope factors, can be better represented comparing the inflation and GDP of these three countries with global level and global slope factors.

To obtain an inflation and GDP annual growth series that represents inflation and GDP annual growth of these three countries together, the PCA will be used. The inflation in the analyzed period is stable in South Korea and Mexico, however in Brazil there was an increase in the inflation because the economic risk of the new government. Because of the influence of Brazilian inflation there is great impact in the level factor. The correlation between the extracted inflation via PCA and global level factor is forty one percent. The GDP annual growth extracted by PCA have good correlation with global slope factor, it was eleven percent in the period.

4.2.1 Variance Decomposition

The variance of specific country factor can be explained as a variation of global and idiosyncratic factors. It is important to evaluate this decomposition because this can explain the magnitude of variations of each factor, and infers the influence of global movements in the country economy.

The formulation of country factor can be extracted from equation (12a and 12b) using a simple definition of variance, hence:

$$var(l_{i,t}) = (\beta_i^l)^2 \times var(L_t) + var(\varepsilon_{i,t}^l)$$

$$var(s_{i,t}) = (\beta_i^s)^2 \times var(S_t) + var(\varepsilon_{i,t}^s)$$

In the Kalman filter estimation, the extraction of the global and idiosyncratic factors may be correlated even if the global and idiosyncratic factors are not correlated. The last specification of factor's variance obligate that these variables are orthogonal, and using the Kalman filter's estimations is not correct to decompose those variances. Hence, the orthogonalization will be obtained regressing the countries factors on the global extracted factor and the results of these estimations will be used in the decomposition formulate.

$$\begin{aligned} l_{i,t} &= c_i^l + \delta_i^l \times L_t^{Global} + \varepsilon_{i,t}^l \\ s_{i,t} &= c_i^s + \delta_i^s \times S_t^{Global} + \varepsilon_{i,t}^s \end{aligned}$$

The resulta are:

Level Factors Volatility			
	Brazil	South Korea	Mexico
Global Factor	44.69%	26.61%	7.29%
Idiosyncratic Factor	55.31%	73.39%	92.71%
Slope Factors Volatility			
	Brazil	South Korea	Mexico
Global Factor	14.00%	0.03%	0.00
Idiosyncratic Factor	86.00%	99.97%	100.00%

Table 6: Variance Decomposition

The influence of global factor in Brazil's level factor is forty four percent. For the level factor the influence of global level factor vary in the range of seven percent to

forty four percent. The influence of global slope factor is less than global level factor. The influences vary in the range of zero to fourteen percent. The interesting result can be viewed in the table. The global level factor volatility influence the volatility of Brazil's yield level more than two other countries, this result are expected because Brazilian economy is more instable than other two. The influence global level in the volatility of Mexico's yield level is lower that other two, this result is expected again, because Mexico is the US proxy and suffer lower influence from external factors, but influence more than the others. From, global slope factor volatility the results are expected again. Only in Brazil the global slope volatility influence, in spite of the vulnerability of Brazilian's economy. In the other two, the influence is zero.

5 Conclusion

The present work has extended the DLY studies about global yield curve that covered the developed countries. The proposal of this study is to extend the seminal work to emerging markets' countries. The perception of emerging market suffers influence of external factors or global factors, is the main argument of this work.

The seminal work of global yield curve extended the parsimonious yield curve proposed by NS and DL to a global environment. That work proposed the hierarchical model in which country yield level and slope factor may depend on the global factors, as well as idiosyncratic factors. This work already uses a monthly data set of Brazil, South Korea and Mexico's zero-coupon bond yield from June of 1998 to September of 2007 and using the McCulloch's (1977) cubic spline to extract the unobserved maturities.

The results indicate, in concordance of DLY that level global factor have large influence in the countries' yield in special in Brazil's yield curve because of the Brazil's vulnerability of external crises, and the global level has a negative tendency that explain the decrease of emerging countries yield in recently years. The opposite result, now in discordance of DLY, about slope global factor can be found. The global slope factor can be interpreted as the sort run tendency of yield, and in developed countries the level of yield are lower than emerging countries and the tendency to increase their yield's level are lower than developed countries, in order to highest level yield.

The results about the influence of global factors variability in country variability obey the stylized facts. The influence of global level factor variability in Brazils' level factor is higher than other two, in order to Brazils' vulnerability, and the global level factor vulnerability influence the Mexico level yield less that other two, in order to Mexico is the US proxy. The results of global slope factor variability again respect the stylized facts, because of the level's yield of emerging countries. The level's yield in develop countries are lower than emerging countries, and because of this the variability is higher than emerging countries.

The results of present work indicate the existence in the emerging markets the global level and global slope factors. These factors pursue an important and significant fraction in the bond yield determination; moreover these can be related, specially global level, with macroeconomics variables, like inflation and GDP annual growth.

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APPENDIX A - CUBIC SPLINE

Spline

The discount function gives the present value of \$1.00 which is repaid in m years. Hence, the correspondent yield to maturity of investment $y(m)$, or *spot interest rate*, or *zero coupon rates*, must satisfy the following equation under continuous compounding:

$$d(m)e^{-y(m)\times m} = 1$$

$$d(m) = e^{-y(m)\times m}$$

The definition of discount function and spline method can be expressed using k continuously differentiable functions $s_j(m)$ to approximate the discount rates:

$$y(m) = a_0 + \sum_{j=1}^k a_j s_j(m) \quad (22a)$$

or

$$d(m) = a_0 + \sum_{j=1}^k a_j s_j(m) \quad (22b)$$

Where $s_j(m)$ are known functions of maturities m , and a are the unknown coefficients to be determined from the data. Since the discount rate must satisfy the constraint $d(0) = 1$, set $a_0 = 1$ and $s_j(0) = 0$ for $j = 1, \dots, k$, and once the functional form of $s(m)$ is determined, the coefficients a can be easily estimated by linear regression.

McCulloch(1971) used the quadratic spline, which uses quadratic polynomial for $s_j(m)$, and McCulloch(1977) used the cubic spline, which uses cubic polynomial. MacCulloch's choice is based on fact that a polynomial function is easy to evaluate, differentiate and integrate.

In Bliss(1996) was tested different methods to evaluate interpolations, and Bliss conclude that Unsmoothed Fama-Bliss(1987) is better than the others, but the difference between it and cubic spline is small, because of this in this work it will be used McCulloch's cubic spline for computational convenience, following Brennan and Xia (2003).

Cubic Spline

Consider these points:

$$(m_0, Y_0), (m_1, Y_1), \dots, (m_{n-1}, Y_{n-1}), (m_n, Y_n).$$

Using this points it is possible to fit cubic spline through the data.

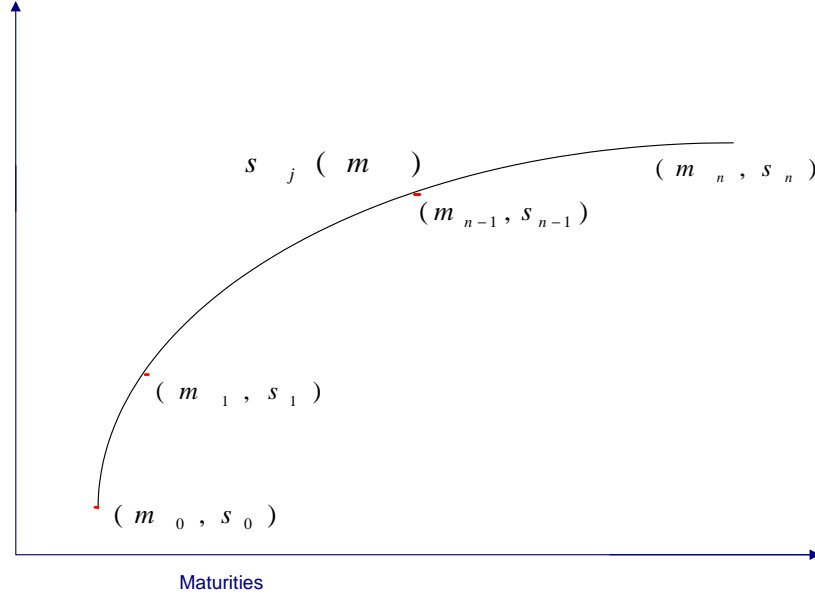


Figure 10: Interpolation of discrete data

The splines are given by:

$$\begin{aligned} s_1(m) &= \varphi_1 m^3 + \rho_1 m^2 + \gamma_1 m + \zeta_1 & m_0 \leq m \leq m_1 \\ s_2(m) &= \varphi_2 m^3 + \rho_2 m^2 + \gamma_2 m + \zeta_2 & m_1 \leq m \leq m_2 \\ &: \\ &: \\ s_n(m) &= \varphi_n m^3 + \rho_n m^2 + \gamma_n m + \zeta_n & m_{n-1} \leq x \leq m_n \end{aligned}$$

The solution for $4n$ coefficients is solved using simultaneous linear equations. The question that arisen is the numbers of equations is the same of variables? There are needs to find $4n$ equations for $4n$ coefficients and it can be made differentiating the splines in different intervals that have the same central point. In cubic spline it must differentiate two times. Even though it must consider $\varphi_1 = 0$ or the first spline as a linear spline, to complete the numbers of equations.

Having these coefficients, one can use equation (22a or 22b) and find the parameters φ .

APPENDIX B - SUB-SAMPLE - PRELIMINARY ANALYSIS

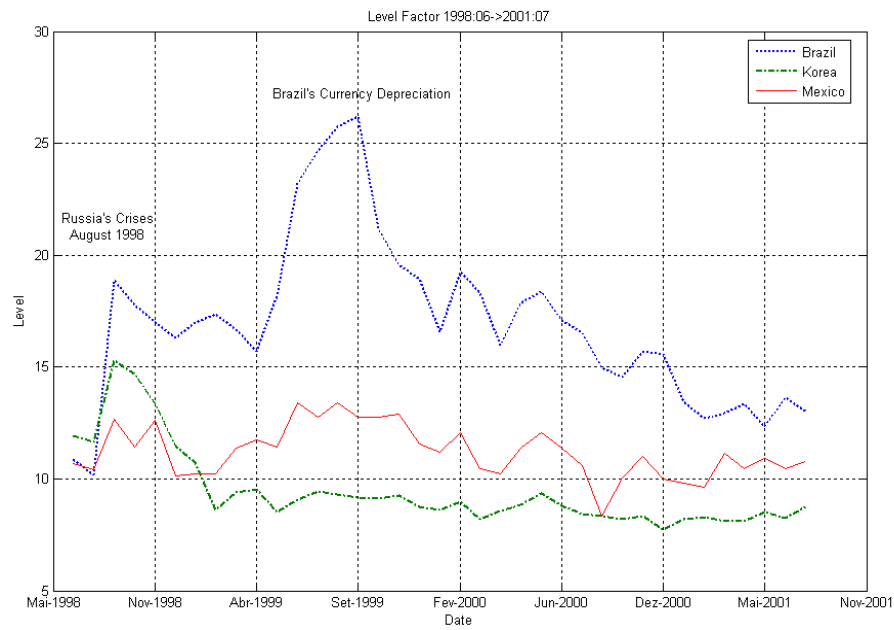


Figure 11: Level Factor 1998:06->2001:07

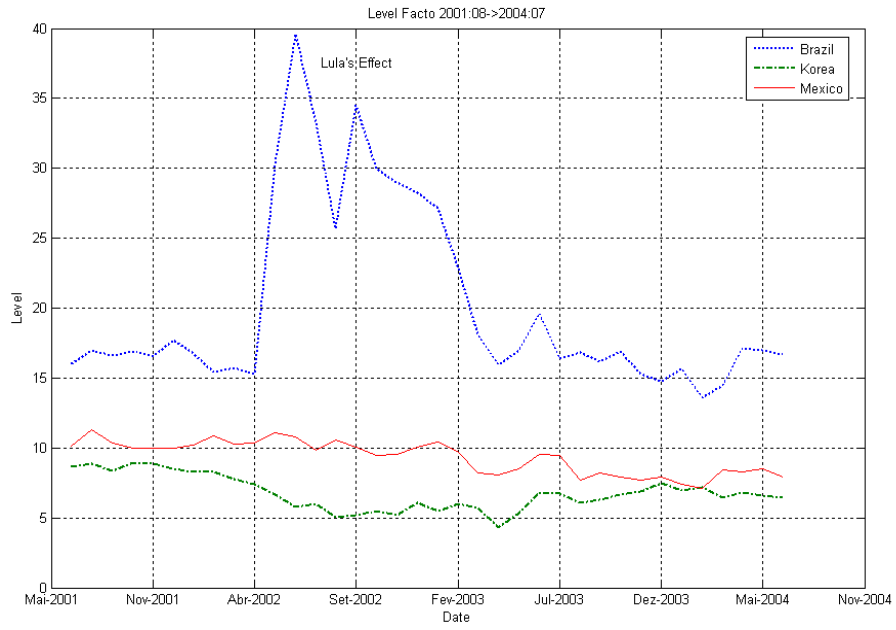


Figure 12: Level Factor 2001:08->2004:07

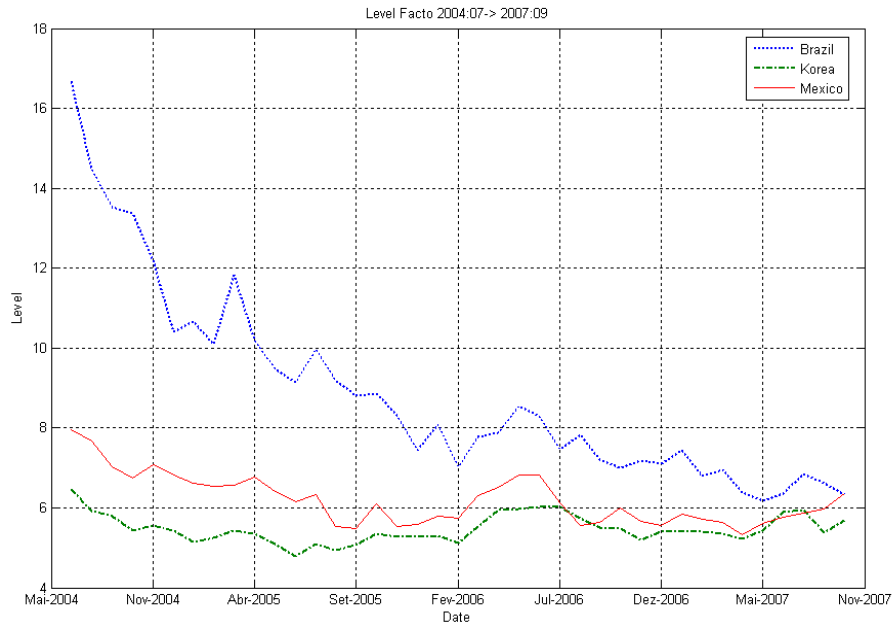


Figure 13: Level Factor 2004:07->2007:09

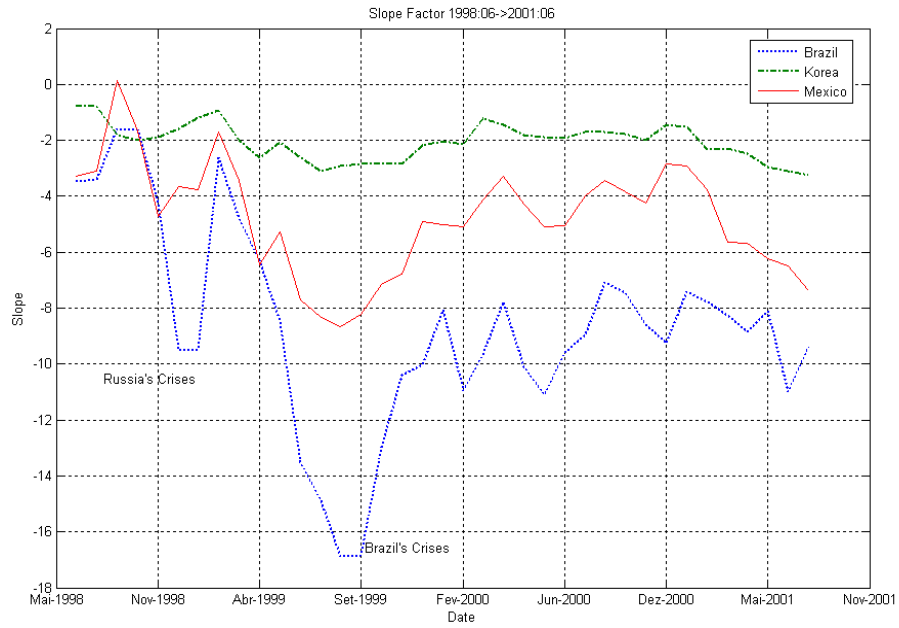


Figure 14: Slope Factor 1998:06->2001:06

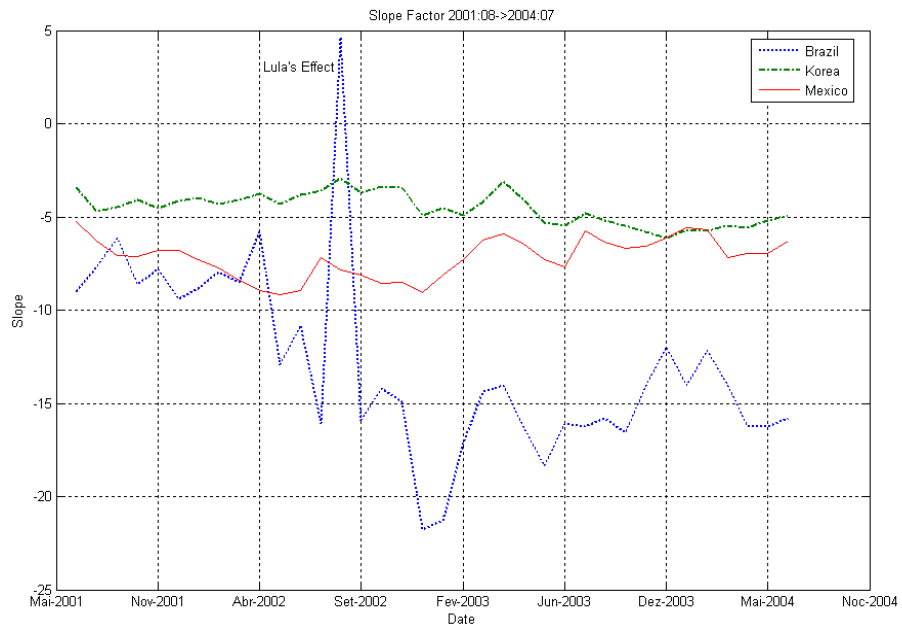


Figure 15: Slope Factor 2001:08->2004:07

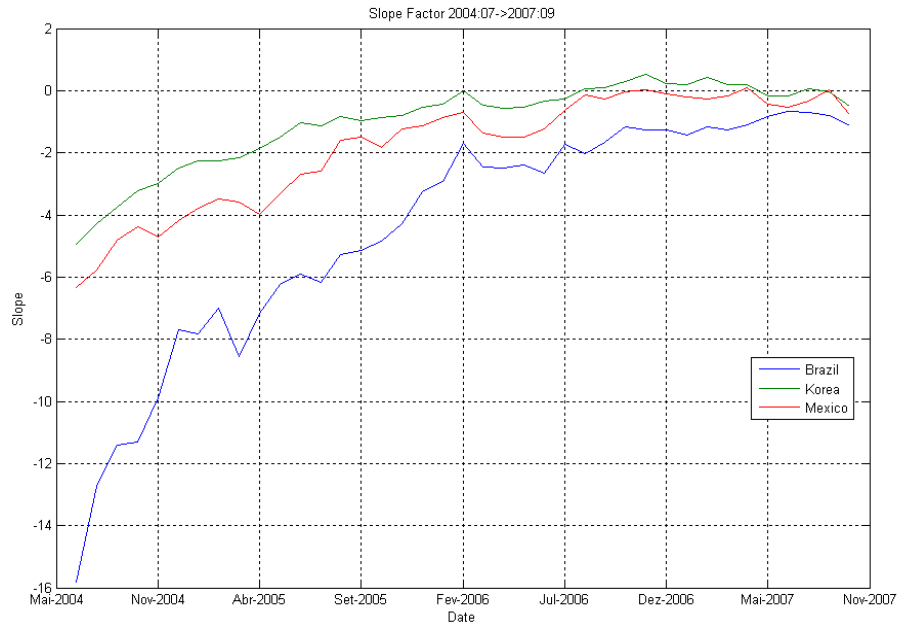


Figure 16: Slope Factor 2004:07->2007:09

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