Escola de Pós-Graduação em Economia - EPGE
Fundação Getulio Vargas

Further investigation of the uncertain trend in U.S. GDP

Dissertação submetida à Escola de Pós-Graduação em Economia
da Fundação Getulio Vargas como requisito para obtenção do
Título de Mestre em Economia

Aluno: Jaime de Jesus Filho

Orientador: Luiz Renato Lima

Rio de Janeiro
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Abstract

The presence of deterministic or stochastic trend in U.S. GDP has been a continuing debate in the literature of macroeconomics. Ben-David and Papell (1995) found evindence in favor of trend stationarity using the secular sample of Maddison (1995). More recently, Murray and Nelson (2000) correctly criticized this finding arguing that the Maddison data are plagued with additive outliers (AO), which bias inference towards stationarity. Hence, they propose to set the secular sample aside and conduct inference using a more homogeneous but shorter time-span post-WWII sample. In this paper we re-visit the Maddison data by employing a test that is robust against AO's. Our results suggest the U.S. GDP can be modeled as a trend stationary process.

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1 Introduction

The presence of deterministic or stochastic trend in the output has been continuing discussion in the literature of time series analysis. The concern devoted to this topic is well deserved. Whether the series can be modeled as stationary fluctuations around a deterministic trend or as difference stationary process is important for many reasons. First, it has important consequences for the long run behavior of output dynamics. A shock to a trend stationary process has a transitory impact, whereas a shock to a difference stationary process shift the trend in a permanent fashion. In particular, if trend is deterministic then one would expect that real shocks, shock that shift permanently the trend, will be small and infrequent. However, if trend is stochastic, then real shocks will be a important source of macroeconomic fluctuations. In this way, real factors are the main source of the economic fluctuations, in agreement with Real Business Cycle theory. Therefore, there would be good reasons to identify and model such shocks. Second, when output is used in regressions with other variables, the interpretation of the regression results is sensitive to the kind of trend in the variables involved. This phenomenon is related to the 'spurious regression ' literature due to Granger and Newbold (1974). Third, as it was pointed in Stock and Watson (1988), the economic forecast is very dependent on the form that the trend is modeled. Trend and difference stationary models imply very different dynamics and hence different point forecasts.

Therefore, it is important to assess carefully the reliability of the unit root hypothesis. Testing for the null hypothesis of unit root against the alternative of trend stationarity using a univariate time series provides a natural framework for investigating the presence of stochastic or deterministic trend in output. Until the seminal work of Nelson and Plosser (1982), the usual representation of the aggregate output in the empirical works was the trend stationary model. In their work, however, they could not reject the unit root null hypothesis against the trend stationary alternative for 13 out of 14 long-term annual U.S. macro series, including the real GNP.

Nelson and Plosser's study was followed by several other works, some of them corroborate the difference stationary model and others reject the null hypothesis of unit root. Campbell and Mankiw (1987) claims that the output fluctuations are permanent. Likewise, Cheung and Chinn (1996) analyze the output of 126 countries and, using two kinds of test, one with the trend stationary model under the null hypothesis and another with difference stationary model under the null, they find great evidence in favor of difference stationary model. Cochrane (1988) finds little evidence of stochastic trend in U.S. GNP. Ben-David and Pappel (1995) have conducted unit root inference using the secular sample tabulated in Maddison (1995), and found strong evidence in favor of trend stationarity for the U.S. output.

A important consideration in the analysis of unit root is that many economic time series data are affected by infrequent but relevant events such as oil shocks, wars, natural disasters and changes in policy regimes (see Balke and Fomby, 1994). These events can be considered as outliers or structural breaks in the data series and can have important effects on the unit root tests. Hence, a lot of effort has been devoted in the development of procedures that are robust to outliers or structural breaks and can be applied to nonstationary time series. In general, there are two approaches to test the unit root hypothesis in the presence of outlying observations. The first one is the so called "intervention analysis " suggested by Box and Tiao (1975). According to their methodology, outlying events or trend breaks can be separated from the noise function and be modeled as change or "interventions " in the deterministic part of the general time series model. Basically, dummies variables are included in the ADF regression at dates that the events occur. Ben-David et al. (2003) use the intervention analysis based approach to test for unit root in 16 industrialized countries using the Maddison's data. They test the null hypothesis of unit root against alternatives of trend stationarity allowing one or two breaks in the trend. Their results show that if there are two breaks in the alternative model, then the null of unit root is rejected for three-quarters of the countries, including the U.S. If only one break is considered in the alternative model, then the null of unit root is rejected for about half of the countries (the U.S. included).

Recent criticism about the intervention-analysis based approach has already emerged. Indeed, as pointed out by Montañés et al. (2005) unit root tests based on intervention analysis are very sensitive to the specification of the alternative model, and this may explain why the findings reported by Ben-David et al. (2003) change so much when they alter the specification of the alternative model from one break to two breaks in the deterministic trend.

Considering these drawbacks in the intervention analysis, we propose to use an alternative approach that is based on robust statistics. Hoek et. al. (1995), show that additive outliers (AO) can bias traditional unit root tests towards stationarity. This result suggests that if one wants to conduct unit root robust inference against AO's, then it is relevant to consider unit root tests based on a robust estimator. They suggest to consider the maximum likelihood estimator based on the student-t distribution (MLT estimator). Robust unit root test based on the MLT estimator has been extensively proposed in the literature. A partial list along this direction includes Lucas (1995a,b), Thompson (2004), and Xiao and Lima (2005). The basic idea is to base inference on a Student-t error distribution rather than the usual Gaussian distribution. An important feature of this robust approach is that there is no need to date outlying observations before implementing the unit root tests. In other words, unlike the intervention-analysis based approach, the unit root tests based on the MLT estimator should not be seen as sequential tests. Rodriguez (2004) argues that when the researcher is only interested in a robust estimate of a specific coefficient, it is correct simply rely on a robust estimator.

In a recent paper, Murray and Nelson (2000) claim that the Maddison data are plagued with AO's. They conduct monte carlo experiments to show that the unit root tests based on least squares estimation tend to bias unit root inference towards stationarity when AO is present. Moreover, they show that additive outliers can signal the presence of permanent shifts in trend that did not occur. As a solution to the AO problem, Murray and Nelson proposed to shift from the annual Maddison data to a more homogenous but shorter time-span sample. They considered WWII era observations and found evidence against the trend stationary model.

Motivated by the potential importance of the presence of unit root in the U.S. output, we re-visit the Maddison data. We use two robust tests to assess the presence of stochastic trend in U.S. GDP: the Lucas' test, based on the MLT estimator that considers errors with Student-t distribution with three degrees of freedom, and the PADF test (Xiao and Lima, 2005), which is also based on the MLT estimator, but that selects the degrees of freedom using a data-dependent procedure. The tests based on the MLT estimator are not sequential procedures and, therefore, identification of the dates of the outliers are not relevant at all. Nonetheless, we use a test to detect AO's in U.S. GDP and our finding is that only one observation can be considered as an outlying observation, that is, the 1933 observation. Finally, the robust unit root analysis suggests that the U.S. real output is trend stationary.

The outline of the paper is as follows: section 2 introduces the concept of influence function and use it to show how AO affects unit root inference. A solution based on MLT estimator is proposed. With the goal of detect additive outliers in U.S. GDP, section 3 presents a test to detect AO's in a time series which is based in Vogelsang (1999). In the section 4, we present two robust unit root tests based on the MLT estimator proposed by Lucas (1995a,b) and, recently, by Xiao and Lima (2005). A new form of computing critical values, based on Thompson (2004), is used . In the section 5, we re-visit the Maddison data applying the AO detection test and then the robust unit root tests. Section 6 concludes.

2 Unit root tests and additive outliers

In this section, we introduce the concept of additive outlier (AO) and show its effect on unit root inference. The discussion presented here is strongly based on the work of Hoek et al. (1995). Suppose that the stochastic process is generated according the following DGP

$$
\begin{aligned}
y_t &= (1 \quad z_t)x_t + z_t w_t, \qquad (1)\\
\phi(L)x_t &= \varepsilon_t,\\
w_t &= x_t + \xi_t,
\end{aligned}
$$

where $\phi(L)$ is a polynomial of order p in the lag operator L[1], z_t is a binary random variable, which is equal to 1 with probability γ and to 0 otherwise, x_t is the outlier-free process, w_t is the contaminating process, and ξ_t is the additive outlier. Therefore, model (1) tells us that the time series will be equal to x_t (the outlier-free process) with probability γ and equal to $x_t + \xi_t$ otherwise.

The effect of additive outliers on unit root inference can be shown through the influence function (IF). Roughly speaking, the IF measures the change in the value of an estimator when an outlier is added to the sample.

As an example, consider the AR(1) process $x_t = \phi x_{t-1} + \varepsilon_t$, with $t = 1, ..., T$, where ε_t is i.i.d. with finite variance. Let $z_t = 0$ for all $t = s$ and $z_t = 1$ for some $1 < s < T$. Hoek et al. (1995) prove that the OLS estimator of ϕ based on the observed series y_t will be equal to

$$
\widehat{\phi}_{ols} = \frac{\sum_{t=1}^{T} y_t y_{t-1}}{\sum_{t=1}^{T} y_{t-1}^2} = \frac{\xi(x_{s-1} + x_{s+1}) + \sum_{t=1}^{T} x_t x_{t-1}}{\xi^2 + 2\xi x_s + \sum_{t=1}^{T} x_{t-1}^2} = O_p(\xi^{-1}). \qquad (2)
$$

Note that ξ represents the additive outlier. Therefore, existence of ξ will make the OLS estimator, $\widehat{\phi}_{ols}$, be biased towards zero. Hence, unit root tests based on OLS estimation, such as the ADF test, will likely overreject the null hypothesis of unit root when AO's are present.

Therefore, it is important to test for unit root taking into account the presence of additive outliers. The first step is to consider another estimator that possesses a bounded IF. A simple choice suggested by Hoek et al. (1995) is the M estimator based on the class of Student-t distributions. This estimator is nothing else than a conditional maximum likelihood estimator within the class of iid Student-t distributed innovations (MLT). The basic idea is to base inference on a Student-t error distribution instead of usual Gaussian distribution.

As an example, consider the AR(1) model $y_t = \phi y_{t-1} + \varepsilon_t$. The MLT estimator of ϕ maximize the following log-likelihood function

$$
L = \text{constant} + \frac{n}{2}\ln\Theta \quad \frac{\nu+1}{2}\sum_{t=j+2}^{n}\ln\left\{1 + \frac{\Theta}{\nu}\left[\varepsilon_t\right]^2\right\}, \qquad (3)
$$

[1] Note that we do not restrict x_t to be stationary, that is, we can also assume that $\phi(L) = 1 - L$

where the parameter Θ measures the spread of the disturbance distribution and ν is the degree of freedom that measures the tail thickness. Large values of ν corresponds to thin tails in distribution. For given parameters ν and Θ, denoting Θ/ν as θ, the MLT of ϕ is the solution of the following optimization problem

$$\min \sum_t \varphi(\varepsilon_t). \tag{4}$$

where $\varphi(\varepsilon_t) = \left(\frac{\nu+1}{2}\right) \ln \left\{1 + \theta \left[\varepsilon_t\right]^2\right\}$.

Hoek et al. (1995) noted that the score function, $\psi(\varepsilon) = \varphi'(\varepsilon)$, of the MLT estimator is $O(\varepsilon_t^{-1})$ and therefore satisfies the necessary condition for the boundness of the influence function in a time series context. The score function of the MLT estimator can be written as

$$\psi(\varepsilon_t) = \left(\frac{\nu+1}{2}\right) \frac{2\theta\varepsilon_t}{1 + \theta \left[\varepsilon_t\right]^2}. \tag{5}$$

In the OLS estimator, the objective function is $\varphi(\varepsilon) = \left[\varepsilon_t\right]^2$, then its score function is linear in the innovation

$$\psi(\varepsilon_t) = 2\varepsilon_t. \tag{6}$$

3 Searching for additive outliers in U.S. GDP

There is a large literature in econometrics on outlier detection in ARMA models. The standard approach is to estimate a fully parameterized ARMA model and construct a t-statistic for the presence of an outlier. Such a t-statistic is constructed at all possible dates and the supremum is taken. The value of the supremum is then compared to a critical value to decide if an outlier is present.

To assess the existence of AO's in U.S. GDP logarithm we employ Vogelsang (1999) procedure to detect outliers. The data generating process is of the following general form:

$$y_t = \mu_t + \sum_{j=1}^{n} \delta_j D\left(T_{ao,j}\right)_t + \varepsilon_t, \tag{7}$$

where $D\left(T_{ao,j}\right)_t = 1$ if $t = T_{ao,j}$ and 0 otherwise. This permits the presence of n additive outliers at dates $T_{ao,j}\left(j = 1, 2..., n\right)$. The term μ_t represents the deterministic components. In our case $\mu_t = \mu$ if the series is non-trending or $\mu_t = \mu + \beta t$ if it is trending. The error term has the following form:

$$\varepsilon_t = \vartheta \varepsilon_{t-1} + \nu_t,$$

where ν_t is a stationary process and the parameter ϑ is set to be one.

The detection procedure starts with the following regression estimated by OLS:

$$y_t = \widehat{\mu}_t + \widehat{\delta} D\left(T_{ao,j}\right)_t + \widehat{\varepsilon}_t, \tag{8}$$

where $D\left(T_{ao,j}\right)_t = 1$ if $t = T_{ao}$ and 0 otherwise.

Let $t_{\widehat{\delta}}\left(T_{ao}\right)$ denote the t-statistic for testing $\delta = 0$ in (8). Following Chen an Liu (1993), the presence of an additive outlier can be tested using:

$$\tau = \sup_{T_{ao}} t_{\widehat{\delta}}\left(T_{ao}\right) \ . \tag{9}$$

Assuming that $\lambda = T_{ao}/T$ remains fixed as T grows, Vogelsang (1999) showed that as T ,

$$t_{\widehat{\delta}}\left(T_{ao}\right) \quad H\left(\lambda\right) = \frac{W^{*}(\lambda)}{\left(\int_0^1 W^{*}(r)^2 dr\right)^{1/2}},$$

where $W^{*}\left(\lambda\right)$ denotes a demeaned (or detrended) standard Wiener process. The asymptotic critical values for τ were obtained using simulations.

The outlier detection procedure recommended by Vogelsang (1999) is put into practice as follows. First, the τ statistic is computed for the entire series and τ is compared to the respective critical value. If τ exceeds the critical value, then an outlier is detected at date

$$\widehat{T}_{ao} = \arg\max_{T_{ao}} \ t_{\widehat{\delta}}\left(T_{ao}\right) \ . \tag{10}$$

The outlier and the corresponding row of the regression is dropped and (8) is again estimated and tested for the presence of another outlier. This continues until the test does not reject H_0 : the date T_{ao} is not an outlier.

A limitation of above procedure is that when we apply it in an iterative fashion to search for many outliers, it has grave size distortions. The reason for this is that the limiting distribution of the τ test is only valid in the first step of the iteration (see Theorem 1 in Perron and Rodriguez, 2003). In subsequent steps, the asymptotic critical values need to be modified. Perron and Rodriguez (2003) specify that the appropriate critical values to be used in the full iterative procedure will be a_i, where a is the level of significance and i index the step in the full iterative process. In the empirical application, they suggest to use, for a 5% test, the following critical values: 3.33, 4.86, 13.16 and 18.20 for the first, second, third and fourth steps, respectively. Notice that the 5% critical value in the second step is different from the one used by Vogelsang (1999).

In the next section, we introduce unit root tests based on the MLT estimator, which will turn out to be robust against AO's . The robust tests are not sequential procedures and, therefore, identification of the dates of the outliers are not relevant at all. Hence, the application of the aforementioned procedure of outlier detection will play (in our empirical analysis) an illustrative role only. The detection of outliers help us to confirm visual inspections, allowing us to relate macroeconomic events to the dates of occurrence of outliers.

4 Robust unit root tests

As showed in section 2, the sensitivity of the OLS estimator to AO's leads the Augmented Dickey and Fuller t-test to overreject the null hypothesis of unit root. To solve this problem, an estimator less sensitive to outliers must be used. The robust estimator is used to construct robust version of the ADF test statistic. In particular, Hampel et al. (1986) and Peracchi (1991) showed that the robustness properties of estimators carry over to test statistics based on such estimators.

Huber (1964, 1973) introduced a class of M estimators for the location problem in order to obtain more robust estimators which generally have good properties over a wide range of distributions. The M-estimators are obtained from solving the extreme problem like (4) by replacing the quadratic criterion function in OLS estimation with some general criterion function $\varphi(.)$. In the case that $\varphi(.)$ is the true log density function of the residuals, the M-estimator is the maximum likelihood estimator. In the next subsections we will describe the robust unit root test proposed by Lucas (1995a,b), which is based on MLT estimator. The computation of correct asymptotic critical values are also discussed and the last subsection describes the robust unit root test of Xiao and Lima (2005) which is also based on M estimator but they use a data-dependent procedure to select the appropriate criterion function.

4.1 Lucas ' robust test

From the class of M estimators, Lucas (1995a) specifies an particular element of it, namely the pseudo maximum likelihood estimator based on the Student t distribution (MLT estimator). This estimator is often employed in the econometric literature as first relaxation of the usual normality assumption. The distributions of the innovations is set to be a Student t(3) (with three degrees of freedom).

Let y_t be a time series whose largest autoregressive root, α, is close to unity:

$$y_t = \alpha y_{t-1} + u_t. \tag{11}$$

The residual term u_t is serially correlated. In the above model, the autoregressive coefficient α plays an important role in measuring persistency in economic and financial time series. Under regularity conditions, if $\alpha = 1$, y_t contains a unit root and is persistent; and if $\alpha_1 < 1$, y_t is stationary.

Following Dickey and Fuller (1979), u_t is parameterized as a stationary $AR(k)$ process

$$A(L)u_t = \varepsilon_t, \tag{12}$$

where $A(L) = \sum_{i=0}^{k} a_i L^i$ is a k-th order polynomial of the lag operator L, $a_0 = 1$, and ε_t is an iid sequence. Combining (11) and (12), one obtains the well-known Augmented Dickey-Fuller (ADF) regression model

$$\Delta y_t = \gamma' x_t + \rho y_{t-1} + \sum_{j=1}^{k} \psi_j \Delta y_{t-j} + \varepsilon_t. \tag{13}$$

where x_t is a deterministic component of known form and γ is a vector of unknown parameters. The leading cases of the deterministic component are (i) a constant term $x_t = 1$, and (ii) a linear time trend $x_t = (1, t)'$.

One wants to test the unit root hypothesis ($\rho = 0$) based on estimators of ρ. In the simple case where ε_t is normally distributed, given observations on y_t, the maximum likelihood estimators of ρ and $\{\psi_j\}_{j=1}^{k}$ are simply the least squares estimators obtained by minimizing the residual sum of squares.

In the case where ε_t has Student t distribution, putting (13) in (3), the M estimator of $(\gamma', \rho, \{\psi_j\}_{j=1}^{k}) = \Pi$ solves the following first order conditions:

$$\sum_{t=j+2}^{n} \left(1 + \frac{\Theta}{\nu} \left[\Delta y_t \quad \gamma' x_t \quad \rho y_{t-1} \quad \sum_{j=1}^{k} \psi_j \Delta y_{t-j} \right]^2 \right)^{-1} \frac{\partial \varepsilon_t}{\partial \Pi} = 0. \tag{14}$$

If $\widehat{\rho}$ is M estimator of ρ, then the t-ratio statistic of $\widehat{\rho}$ is:

$$t_{\widehat{\rho}} = \frac{\widehat{\rho}}{se(\widehat{\rho})}. \tag{15}$$

Lucas (1995a) estimate the covariance matrix by

$$\widehat{\Omega} = \left[\sum_{t=1}^{n} \varphi'' (\widehat{\varepsilon}_t) Z_t Z_t' \right]^{-1} \left[\sum_{t=1}^{n} \varphi' (\widehat{\varepsilon}_t)^2 Z_t Z_t' \right] \left[\sum_{t=1}^{n} \varphi'' (\widehat{\varepsilon}_t) Z_t Z_t' \right]^{-1}. \tag{16}$$

This is a heteroskedasticity consistent type covariance matrix estimator as in White (1980), in which was used the abbreviation $(x_t', y_{t-1}, \Delta y_{t-1}, \quad , \Delta y_{t-k})' = Z_t$. This can easily be seen by inserting the $\varphi'(\widehat{\varepsilon}_t) = \widehat{\varepsilon}_t$ function of the OLS estimator. Thus, $t_{\widehat{\rho}}$ is simply the M regression counterpart of the well-known ADF (t-ratio) test for the unit root hypothesis.

It is showed in Lucas (1995b) and in Xiao and Lima (2005) that $t_{\widehat{\rho}}$ has the following limiting distribution

$$t_{\widehat{\rho}} \quad \sqrt{1 \quad \lambda^2} N(0,1) + \lambda \left(\int \underline{W}_1(r)^2 dr \right)^{-1/2} \int \underline{W}_1 dW_1, \tag{17}$$

where $\underline{W}_1$ is a demeaned (detrended) Brownian motion and the weights are determined by λ,

$$\lambda^2 = \frac{\sigma_{u\varphi}^2}{\omega_\varphi^2 \omega_u^2}, \tag{18}$$

where ω_u^2 is the long-run (zero frequency) variance of u_t, ω_φ^2 is the long-run variance of $\varphi'(\varepsilon_t)$, and $\sigma_{u\varphi}(\tau)$ is the long-run covariance of u_t and $\varphi'(\varepsilon_t)$, thus λ^2 is simply the squared long-run correlation coefficient between u_t and $\varphi'(\varepsilon_t)$ .This parameter gives an indication of the discrepancy between the asymptotic distributions of the unit root tests based on M estimators and on the OLS estimator (Lucas, 1995a). Note that when λ goes to 1 we have the Dickey-Fuller's limiting distribution. Departures from Gaussianity are characterized by $0 < \lambda^2 < 1$.

Two things must be observed here: (i) - the Student t distribution is indexed by the degrees of freedom ν which is to be chosen in an arbitrary way; (ii) - the limiting distribution of $t_{\widehat{\rho}}$ depends of a nuisance parameter λ. Lucas (1995a,b) considers $\nu = 3$ to estimate the ADF regression (13) but computes critical values using a wrong approach as suggested in the next subsection.

4.2 Obtaining correct critical values

In computing critical values, Lucas (1995b) simulates the data by generating a time series from a data generating process with i.i.d. innovations drawn of a standard normal distribution. Thompson (2004) showed that the Lucas ' approach is asymptotically incorrect unless the errors are in fact normal. Based on this finding, Thompson (2004) proposed a new method to compute critical values based on a polynomial approximation which leads to correct critical values.

Given the parameter λ, the limiting distribution of $t_{\widehat{\rho}}$ can be approximated by using a polynomial approximation as proposed by Thompson (2004). This approach consist simply in approximating critical values by a third order polynomial in $(1 \quad \lambda)$. Thus, let $q(a, \lambda)$ be the critical value for the a^{th} quantile, so it can be written as

$$q(a,\lambda) \quad x'_{\lambda}\Lambda_a, \tag{19}$$

where $x_{\lambda} = \left(1, (1 \quad \lambda), (1 \quad \lambda)^2, (1 \quad \lambda)^3\right)$ and Λ_a is a 4-dimensional coefficient vector. So the test rejects the null at level a when the statistic is less than $x'_{\lambda}\Lambda_a$. The estimated coefficients for the third order polynomial approximation are provided in Thompson (2004, pp. 9).

This approach is more suitable because it shows that the parameter λ (which depends on the error density) affects the computation of the critical values. Thompson (2004) show that if one tests the null hypothesis of unit root using the Lucas ' test, which assumes a t(3) distribution to estimate the ADF regression but computes critical values assuming a Gaussian distribution, then the 5% critical value is 3.42. When a t(3) is used to both estimation of the ADF regression and computation of critical values, Thompson (2004) reports a 5% critical value of 3.01. Consequently, the Lucas ' test has less power - against the trend stationary model - than other robust tests that consider the t(3) distribution for both estimation of the ADF regression and computation of critical values using (19).

4.3 The partially adaptive test (PADF)

We have just discussed that the parameter λ depends on the error density and, therefore, the Lucas' method will give incorrect asymptotic critical values for the robust test unless the errors are in fact normal. Likewise, the approach developed by Thompson (2004) gives incorrect critical values unless the true innovation distribution is equal to the assumed distribution. For example, if the true distribution is a student-t with 9 degrees of freedom and one computes critical values assuming a student-t with 3 degrees of freedom, then the Thompson approach will give imprecise critical values since its polynomial approximation will be based on the value of λ for t(3) rather than for a t(9) distribution. In this section, we consider a partially adaptive approach that tends to give correct critical values because it approximates the true distribution by the data distribution and uses the latter to estimate λ and then the critical values.

Xiao and Lima (2005) use a data-dependent procedure to select an appropriate criterion function for the estimation. In other words, they consider the partially adaptive estimator based on the family of Student-t distributions introduced by Postcher and Prucha (1986).

They argue that the choice of this family is based on two facts: (i) the family of Student-t represents an important dimension of the space of distributions, including the normal distribution as a limiting case and the Cauchy distribution as a special case. Its adaptation parameter will depend on the scale and thickness parameters ($v =$ degrees of freedom), which can be easily estimated from the data using the approach proposed by Potscher and Prucha (1986) and; (ii) unlike the

OLS estimator, the M-estimator based on Student-t distribution has a bounded influence function, meaning that it is less sensitive to the presence of additive outliers as it was mentioned in the previous sections.

The difference between the approach proposed by Xiao and Lima (2005) and the one proposed by Lucas (1995b) and Thompson (2004) is that instead of select an arbitrary value for the degrees of freedom, they estimate an value for v from the data (data-dependent procedure). Potscher and Prucha (1986) discussed the estimation of the adaptation parameters v and Θ. In particular, if we denote $E\left(u_t{}^k \right)$ as σ_k, then for $v > 2$, we have:

$$\frac{\sigma_2}{\sigma_1^2} = \frac{\pi}{v \quad 2} \frac{\Gamma\left[v/2\right]^2}{\Gamma\left[(v \quad 1)/2\right]^2} = d(v) \tag{20}$$

and

$$\Theta = \frac{1}{\pi} \frac{v\Gamma\left[(v \quad 1)/2\right]^2}{\sigma_1^2 \Gamma\left[v/2\right]^2} = q(v, \sigma_1) \tag{21}$$

Potscher and Prucha (1986) show that $d(.)$ is analytic and monotonically decreasing on $(2, \quad)$ with $d(2) = \quad$ and $d(\quad) = \pi/2$. Thus, given estimator of σ_1 and σ_2, v can be estimated by inverting $d(v)$ in (20) and thus an estimator of θ can be obtained from

$$\widehat{\theta} = \frac{q(v, \sigma_1)}{\widehat{v}} = \frac{1}{\pi} \frac{\Gamma\left[(\widehat{v} \quad 1)/2\right]^2}{\widehat{\sigma}_1^2 \Gamma\left[\widehat{v}/2\right]^2} \tag{22}$$

For the estimation of σ_1 and σ_2, it is used the sample moments

$$\widehat{\sigma}_k = \frac{1}{n} \sum_t \widehat{\varepsilon}_t{}^k. \tag{23}$$

Notice that $d(.)$ is monotonically decreasing and v and θ can be estimated numerically. Then, in the PADF test $\widehat{v}$ and $\widehat{\theta}$ are used to estimate ρ. The t-ratio statistic in the PADF test, is the same as in (15).

In order to identify the critical value, one needs to estimate λ^2. Xiao and Lima (2005) estimate ω_φ^2, ω_u^2 and $\sigma_{u\varphi}$ parametrically and thus obtain

$$\widehat{\lambda}^2 = \frac{\widehat{\sigma}_{u\varphi}^2}{\widehat{\omega}_\varphi^2 \widehat{\omega}_u^2}$$

Using the estimate of λ^2, they use (19) to select the appropriate critical values and test the unit root hypothesis.

Monte carlo simulations conducted in Xiao and Lima (2005) show that, in terms of power, the PADF dominates the ADF test in the presence of nonGaussianity and it is as powerful as the ADF test when the distribution is normal. The PADF test also perfoms better than other robust tests under misspecification of the criterion function, and perfoms as good as when the criterion function coincides with the true likelihood function. This happens because the PADF test is based on a data-dependent procedure that chooses an adequate criterion function. In sum, there are two reasons by which the PADF test is adequate to test the null hypothesis of stochastic trend in real U.S. GDP. First, it is based on a M-estimator that possesses bounded influence function and, therefore, it is robust against additive outliers. Second, since the criterion function is selected from the data, the effects of the distribution misspecification can be minimized.

In our empirical exercise we will compute four robust test statistics to assess the presence of the unit root in U.S. GDP. The tests are

1 the L_3 test:

It consists in estimating the equation (13) by maximum likelihood (ML) assuming Student-t distribution with three degrees of freedom - t(3) - and then computing the t-ratio (15). Critical values are coming from the polynomial approximation with $\lambda = 0.98$[2]. This is the robust test proposed by Lucas (1995a,b);

2 the T_3 test:

The ADF (13) regression is estimated by ML assuming a t(3) distribution and the t-ratio is computed according to (15). In this case, however, the critical values are computed by the polynomial approximation assuming a t(3) distribution for the innovations. This is the idea for computation of correct critical values developed in Thompson (2004);

3 the T_9 test:

This is just like the T_3 test with the t(3) distribution replaced by a Student-t distribution with nine degrees of freedom.

4 the $PADF$ test:

The $PADF$ test is performed estimating the ADF regression (13) by ML assuming a Student-t($\widehat{v}$) distribution. The parameter v is previously estimated from (22) and (23). The respective t-ratio is calculated and the critical values also come from the polynomial approximation suggested in Thompson (2004) assuming a a Student-t($\widehat{v}$) distribution.

We also compute the non-robust ADF unit root test for comparison purposes.

[2] This is the value of λ for the Gaussian distribution.

5 The uncertain trend in U.S. GDP: new evidence from a robust approach

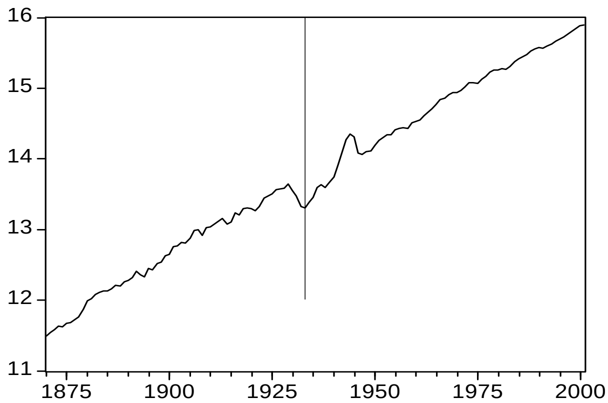
In this section we apply the Vogelsang test to detect outlying observations in the data. Next, we test the null hypothesis of stochastic trend against trend stationarity using the ADF, L_3, T_3, T_9 and $PADF$ tests.

Following Murray and Nelson (2000), we consider the log-transformation of the annual U.S. real GDP series extracted from Maddison (2004)[3]. The time span is of 132 years, with the series ranging from 1870 to 2001, and the base date of the data is 1990.

5.1 Outlier detection

Applying Vogelsang's (1999) procedure to detect outlying observations[4], we found only one outlier. This outlier occured in 1933, year that corresponds to the great depression that followed the great crash of 1929. The following graph display the data series

1 log of U.S.real GDP



The vertical line in the center denotes the outlier position. In 1933, the value of the test statistic is 3.78 (τ statistic in (9)) and the critical value for the first step of the procedure is 3.33 with a level of significance of 5%. In the second step, after discard 1933 observation, the value of the τ statistic is 3.69 but the critical value is 4.86. Observe that if one ignores that the critical value is

[3] All the data are derived from *The World Economy: Historical Statistics*, OECD Development Centre, Paris 2003, which contains detailed source notes.

[4] The routine to perform the outlier detection test was written in Gauss code. To obtain the code you can send an e-mail to authors.

different in each step, he may be induced to find a false outlier. Notice that this finding is opposite of Murray and Nelson (2000) who argue that GDP is plagued with several AO's.

Hoek et al. (1995) show the effect of one outlier on the unit root test. From equation (2) we can see why nonrobust unit root tests tend to overreject the null hypothesis of stochastic trend when an outlying observation is present.

5.2 Results from robust tests

Murray and Nelson (2000) tested for Gaussian behavior in Maddison data and rejected the null hypothesis of normal distribution. They argued that the evidence of nonGaussian behavior was probably provoked by additive outliers characterized by events such as banking crisis and wars. They conduct monte carlo experiments and find that AO bias unit root inference towards stationarity and signal the presence of permanent shifts in trend that did not occur.

Another measure of heavy tails is the thickness parameter of the Student-t distribution, ν. Small ν corresponds to heavy tails and the limiting case, ν , corresponds to the normal distribution. For the real U.S. output we obtained $\widehat{\nu} = 4.71$, suggesting the existence of heavy-tailed distribution in this series. Thus, there is no doubt that U.S. real GDP behavior is inconsistent with linear Gaussian models.

The robust unit root analysis is presented in Table 1. In practice, the lag truncation, k, is chosen by a data dependent procedure. We used the Schwarz information criterion (SIC) and the General-to-specific (GS) strategy, which consist in starting with a maximum value of k chosen a priori, deleting lags sequentially until significance of the 0.10 level. It is well known that SIC will, in the limit, choose the correct k whereas GS will choose k at least as large as the true k, with probability one. Following Murray and Nelson (2000), the a priori maximum lag under GS and SIC is 8.

Considering the model described by (11), (12) and (13), the Table 1 also shows estimates of the autoregressive coefficient α. The first estimate, $\widehat{\alpha}_{L_3}$, is coming from the M regression with Student-t(3) inovations. The second one, $\widehat{\alpha}_{T_3}$, also corresponds to the robust M estimator from the ADF regression when we consider a Student-t(3) distribution, so $\widehat{\alpha}_{L_3} = \widehat{\alpha}_{T_3}$. The third estimates is the M estimator in the case of a Student-t(9). The fourth is the estimates from PADF, when we use a Student-t ($\widehat{v} = 4.71$) Notice that all robust estimates of α seems to be greater than the non robust $\widehat{\alpha}_{ADF}$, suggesting the presence of additive outliers. Not surprisingly, the ADF test strongly reject the null hypothesis of unit root in real GDP (the null is rejected at 1% if k is chosen by SIC and at 5% if it is chosen by GS, see Table 2).

Table 1: Estimates of the autoregressive coefficient

$Lags$	$\widehat{\alpha}_{L_3}$	$\widehat{\alpha}_{T_3}$	$\widehat{\alpha}_{T_9}$	$\widehat{\alpha}_{PADF}$	$\widehat{\alpha}_{ADF}$
$SIC = 1$	0.87	0.87	0.84	0.85	0.81
$GS = 6$	0.81	0.81	0.77	0.78	0.77

Murray and Nelson (2000) correctly criticized this finding arguing that ADF tests are not robust against additive outliers. However, if we use the robust tests, we find that the null hypothesis of unit root is still rejected for almost all tests. Observe in the Table 2[5] that we don't reject the null hypothesis only in the L_3 test for $GS = 6$. This can be explained by the usage of wrong critical value as pointed out by Thompson (2004). Notice that when the correct critical value is used, (which corresponds to the T_3 test), we can reject the null. The same occurs with the T_9 test and $PADF$ tests, in both cases the distribution used to calculate the criterion function is the same used to compute the critical values. Overall, if robust tests with correct critical values are considered, then the null hypothesis of unit root is rejected at 5% if k is chosen by SIC and, at worst, at 10% if k is chosen by GS, suggesting that U.S. real GDP can be described as a trend stationary process with nonGaussian behavior.

Table 2: Value of the test statistics

$Lags$	L_3	T_3	T_9	$PADF$	ADF
$SIC = 1$	-3.50**	-3.50**	-3.41**	-3.40**	-4.24***
$GS = 6$	-2.87	-2.87*	-3.32**	-3.03*	-3.91**

[5] The symbols (*), (**), and (***) indicate rejection of the null hypothesis at 10%, 5%, and 1% level of significance, respectively.

6 Conclusion

The issue about the presence of a unit root in the real U.S. GDP remains open. A lot of effort has been dedicated to this question in the macroeconometrics literature. Whether the time series can be modeled as stationary fluctuations around a deterministic trend or as difference stationary process is an important issue for many reasons, mainly for economic forecast, shock identification, and regression analysis.

In the investigation about the better form to represent the real U.S. GDP, whether by a stochastic trend or by a trend stationary model, one should consider that the data can be affected by infrequent but relevant events such as oil shocks, wars, natural disasters and changes in policy regimes. These events can be considered as outliers or structural breaks in the data series and can have important effects on the unit root tests.

The data on U.S. real GDP reported in Maddison (2004) are contaminated by only one additive outlier, 1933 observation, which, as it was showed, bias unit root inference towards stationarity. This paper re-visited the Maddison sample using a robust approach, which is based on M estimation of the ADF regression assuming that the inovations have a Student-t distribution. We consider two classes of robust test. In the first one, the degrees of freedom of the student-t distribution is assumed to be known a priori. In the second one, we use a data-dependent procedure to estimate the degrees of freedom of the Student-t distribution. In both cases, the asymptotic critical values of the test statistics were obtained by polynomial approximation as suggested by Thompson (2004).

We found evidence in favor of trend stationarity in U.S. real GDP when additive outliers are accounted for. Our results not only suggest that further investigations on the uncertain trend in U.S. GDP are necessary to accumulate more evidence on the empirical relevance of this hypothesis, but also emphasizes that new studies on this topic (or related topics) should consider the use of robust methods that do not neglect the existence of nonGaussian behavior in the U.S. output.

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